34. **On the Periods of Enriques Surfaces. I**

By Eiji Horikawa

University of Tokyo

(Communicated by Kunihiko Kodaira, M. J. A., March 12, 1977)

§ 1. Introduction. A non-singular algebraic surface $S$ is called an Enriques surface if the following two conditions are satisfied:

(i) The geometric genus and the irregularity both vanish.

(ii) If $K$ is a canonical divisor on $S$, $2K$ is linearly equivalent to $0$.

Historically speaking Enriques surfaces were the first example of non-rational algebraic surfaces which satisfy the above condition (i). In this paper we are mainly interested in Enriques surfaces over the field of complex numbers $C$.

From the condition (ii), it follows that there exists a two-sheeted unramified covering $\pi: T \to S$ such that $T$ is a $K3$ surface. Since every $K3$ surface is known to be simply-connected by Kodaira [6], $T$ is the universal covering of $S$. We take a holomorphic 2-form $\psi$ on $T$ which is non-zero everywhere, and consider the integrals

$$\int_{\gamma} \psi \quad \text{for} \quad \gamma \in H_2(T, \mathbb{Z}).$$

We let $\tau$ denote the covering transformation $T \to T$ over $S$ so that $\tau^2 = \text{id}$. Since $S$ has no holomorphic 2-form, we have $\tau^* \psi = -\psi$. On the other hand, $\tau$ acts on $H_2(T, \mathbb{Z})$ as an involution. If $\gamma$ is invariant by $\tau$, then the above integral (1) vanishes. Therefore the periods of $\psi$ are determined by the integrals (1) over those 2-cycles $\gamma$ satisfying $\tau \gamma = -\gamma$. Our main result is that the isomorphism class of $S$ is uniquely determined by these periods. A more precise statement will be given in § 4. Details will be published elsewhere.

§ 2. Elliptic surfaces of index 2. It is known that an Enriques surface $S$ has a structure of an elliptic surface (see [1], [8]). That is, there exists a surjective holomorphic map $g: S \to \mathbb{P}_1$ whose general fibre $C$ is an elliptic curve. Moreover there exists a divisor $G$ on $S$ with $CG=2$. From Kodaira's formula for the canonical bundles of elliptic surfaces ([6], p. 772), it follows that $g$ has two multiple fibres, both being of multiplicity 2. We view $S$ as an elliptic curve over the function field of $\mathbb{P}_1$. Then $G$ is a divisor of degree 2 on this curve. Hence $G$ defines a rational map $f_1$ of degree 2 of $S$ onto a rational ruled surface $W_1$. This map induces, for each generic fibre $C$, a double covering $C \to \mathbb{P}_1$ which is ramified at 4 points. Let $B_1$ be the branch