34. *On the Periods of Enriques Surfaces. I*

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(Communicated by Kunihiko KODAIRA, M. J. A., March 12, 1977)

§ 1. **Introduction.** A non-singular algebraic surface S is called an Enriques surface if the following two conditions are satisfied:

(i) The geometric genus and the irregularity both vanish.

(ii) If K is a canonical divisor on S, $2K$ is linearly equivalent to 0.

Historically speaking Enriques surfaces were the first example of non-rational algebraic surfaces which satisfy the above condition (i). In this paper we are mainly interested in Enriques surfaces over the field of complex numbers $\boldsymbol{C}$.

From the condition (ii), it follows that there exists a two-sheeted unramified covering $\pi: T \to S$ such that T is a $K3$ surface. Since every $K3$ surface is known to be simply-connected by Kodaira [6], T is the universal covering of S. We take a holomorphic 2-form ψ on T which is non-zero everywhere, and consider the integrals

$$(1) \qquad \int_\gamma \psi \qquad \text{for } \gamma \in H_2(T, \boldsymbol{Z}).$$

We let τ denote the covering transformation $T \to T$ over S so that $\tau^2 = \mathrm{id}$. Since S has no holomorphic 2-form, we have $\tau^*\psi = -\psi$. On the other hand, τ acts on $H_2(T, \boldsymbol{Z})$ as an involution. If γ is invariant by τ, then the above integral (1) vanishes. Therefore the periods of ψ are determined by the integrals (1) over those 2-cycles γ satisfying $\tau\gamma = -\gamma$. Our main result is that the isomorphism class of S is uniquely determined by these periods. A more precise statement will be given in § 4. Details will be published elsewhere.

§ 2. **Elliptic surfaces of index 2.** It is known that an Enriques surface S has a structure of an elliptic surface (see [1], [8]). That is, there exists a surjective holomorphic map $g: S \to \boldsymbol{P}^1$ whose general fibre C is an elliptic curve. Moreover there exists a divisor G on S with $CG = 2$. From Kodaira's formula for the canonical bundles of elliptic surfaces ([6], p. 772), it follows that g has two multiple fibres, both being of multiplicity 2. We view S as an elliptic curve over the function field of $\boldsymbol{P}^1$. Then G is a divisor of degree 2 on this curve. Hence G defines a rational map f_1 of degree 2 of S onto a rational ruled surface W_1. This map induces, for each generic fibre C, a double covering $C \to \boldsymbol{P}^1$ which is ramified at 4 points. Let B_1 be the branch