31. *A Remark on the Dimensional Fullvaluedness*

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1. Let $\mathcal{P}$ be the class of paracompact Hausdorff spaces. In this note we investigate a dimensionally fullvalued compactum for $\mathcal{P}$, that is, a compactum X such that $\dim(X\times Y)=\dim X+\dim Y$ for every $Y\in\mathcal{P}$. In particular, we shall show that every movable curve is dimensionally fullvalued for $\mathcal{P}$. This gives a partial answer to [8, Problem 9] or [14, Problem 42-5]. Throughout the paper every group is abelian.

2. Using the same argument as in the proof of [6, Lemma 2] the following is a consequence of [2, Theorem C].

Proposition 1. *Let G be a group and X a compactum with* $\dim X<\infty$ *such that $\check{H}^n(X,A:G)$ contains a copy of G as a direct summand for some closed G_δ set A of X. Then for every $Y\in\mathcal{P}$ with* $\dim Y<\infty$ *$D(X\times Y:G)\leqq n+D(Y:G)$. Here $\check{H}^*$ is the Čech cohomology and $D(X:G)$ is the cohomological dimension of X with respect to G* (cf. [14]).

A space X is said to be *G-cyclic* if $\check{H}^n(X:G)\neq 0$ for $n=\dim X$.

Theorem 1. *Let Q be a divisible abelian group and let X be a Q-cyclic compact metric space which is movable in the sense of Borsuk* [3]. *Then for every space $Y\in\mathcal{P}$ with* $\dim Y<\infty$ *and for every group G the relation $D(X\times Y:G)\geqq\dim X+D(Y:G)$ holds.*

Proof. By [15, 6. 8. 11] we have $\check{H}^n(X:Z)\otimes Q=\check{H}^n(X:Q)\neq 0$, where Z is the additive group of integers. Since Q is divisible, $\check{H}^n(X:Z)/\mathrm{Tor}\,(\check{H}^n(X:Z))\neq 0$, where $\mathrm{Tor}\,(H)$ is the torsion subgroup of H. By [5, Theorem 4.4], we can know that $\check{H}^n(X:Z)/\mathrm{Tor}\,(\check{H}^n(X:Z))$ has property L in the sense of Pontrjagin. Hence by [4, 13.1] $\check{H}^n(X:Z)/\mathrm{Tor}\,(\check{H}^n(X:Z))$ is a free abelian group, because $\check{H}^n(X:Z)$ is countable. Therefore Z is a direct summand of $\check{H}^n(X:Z)$. Since $\check{H}^n(X:G)=\check{H}^n(X:Z)\otimes G$, $\check{H}^n(X:G)$ contains a copy of G as a direct summand. The theorem follows from Proposition 1.

Corollary 1. *If Q is a divisible group, then every Q-cyclic movable compact metric space X is dimensionally fullvalued for $\mathcal{P}$.*

From [14, Theorems 40–7 and 40–8] it is known that the movability of X can not be omitted in Theorem 1 and Corollary 1.

Theorem 2. *Let X be a compact space such that* $0<\dim X<\infty$ *and $\check{H}^1(X:Z)=0$. Then for every group G and for every space $Y\in\mathcal{P}$ with* $\dim Y<\infty$ *the relation $D(X\times Y:G)\geqq D(Y:G)+1$ holds.*