29. On Deformations of Compactifiable Complex Manifolds

By Yujiro KAWAMATA
Department of Mathematics, University of Tokyo

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In this note we shall extend the deformation theory of compact complex manifolds to compactifiable ones defined below.

1. We fix our notation.

$\bar{X}$: a compact complex manifold,

$\bar{D}$: a closed analytic subset of $\bar{X}$ (not necessarily reduced),

$X := \bar{X} - \bar{D}$,

$I_{\bar{D}}$: the ideal sheaf of $\bar{D}$ in $\mathcal{O}_{\bar{X}}$,

$T_{\bar{X}}(\log \bar{D})$: the subsheaf of the tangent sheaf $T_{\bar{X}}$ consisting of derivations of $\mathcal{O}_{\bar{X}}$ which send $I_{\bar{D}}$ into itself.

$\bar{D}$ is said to be of simple normal crossing if (1) $\bar{D} = \bigcup_{i=1}^{h} \bar{D}_i$ where the $\bar{D}_i$ are complex submanifolds of $\bar{X}$, and (2) for each $p \in \bar{X}$, there exist a neighborhood U of p and a system of local coordinates $\{z_1, \cdots, z_n\}$ on U such that $\bar{D}_i = \{z_{r_i+1} = \cdots = z_{r_{i+1}} = 0\}$ for $1 \leqslant i \leqslant h$, where the r_i are integers such that $-1 \leqslant i \leqslant n$ and $r_i \leqslant r_j$ if $i \leqslant j$ and we put $z_0 = 1$ by convention. In that case $\bar{X}$ is called a non-singular compactification of X and $(X, \bar{X}, \bar{D})$ is called *a non-singular triple*. For a fixed X, a bimeromorphic equivalence class $\mathfrak{m}$ of non-singular compactifications of X is called *a meromorphic structure* of X. A pair $(X, \mathfrak{m})$ is called a *compactifiable complex manifold.*

By a family of *logarithmic deformations* of a non-singular triple we mean a 7-tuple $\mathcal{F} = (\mathfrak{X}, \bar{\mathfrak{X}}, \bar{\mathfrak{D}}, \pi, S, s_0, \bar{\psi})$ such that (1) $\pi : \bar{\mathfrak{X}} \to S$ is a proper smooth morphism of (not necessarily reduced) complex spaces $\bar{\mathfrak{X}}$ and S, (2) $\bar{\mathfrak{D}}$ is a closed analytic subset of $\bar{\mathfrak{X}}$ and $\mathfrak{X} = \bar{\mathfrak{X}} - \bar{\mathfrak{D}}$, (3) $\bar{\psi} : \bar{X} \to \pi^{-1}(s_0)$ is an isomorphism such that $\bar{\psi}(X) = \pi^{-1}(s_0) \cap \mathfrak{X}$, and (4) π is locally a projection of a product space as well as the restriction of it to $\bar{\mathfrak{D}}$. A family of *compactifiable deformations* of a compactifiable complex manifold $(X, \mathfrak{m})$ is a 5-tuple $(\mathfrak{X}, \pi, S, s_0, \psi)$ obtained from the 7-tuple above.

Theorem 1. *We have the following exact sequences:*

$$(1) \qquad 0 \longrightarrow T_{\bar{X}}(-\bar{D}) \longrightarrow T_{\bar{X}}(\log \bar{D}) \longrightarrow T_{\bar{D}} \longrightarrow 0$$

where $T_{\bar{D}}$ is the sheaf of derivations $\mathrm{Der}_{\mathcal{O}_{\bar{D}}}(\mathcal{O}_{\bar{D}}, \mathcal{O}_{\bar{D}})$.

$$(2) \qquad 0 \longrightarrow T_{\bar{X}}(\log \bar{D}) \longrightarrow T_{\bar{X}} \longrightarrow N_{\bar{D}} \longrightarrow 0$$

where $N_{\bar{D}} = \mathrm{Coker}\,(T_{\bar{D}} \to T_{\bar{X}} \otimes_{\mathcal{O}_{\bar{X}}} \mathcal{O}_{\bar{D}})$.