28. *Classification of Algebraic Varieties*

By Shigeru IITAKA
Department of Mathematics, Faculty of Science,
University of Tokyo, Hongo, Tokyo 113

(Communicated by Kunihiko KODAIRA, M. J. A., Feb. 12, 1977)

§ 0. **Introduction.** The classification of projective algebraic surfaces was obtained by G. Castelnuovo and F. Enriques at the beginning of this century. Recently, a number of mathematicians have started to develop the classification theory of complete algebraic varieties up to birational equivalence.

Our purpose here is to introduce the classification theory of arbitrary (possibly incomplete) algebraic varieties up to a suitable kind of birational equivalence. When dealing with incomplete algebraic varieties, birational equivalence seems too loose but isomorphism in the category of schemes is too tight. For instance G_a and G_m are birationally equivalent to $\boldsymbol{P}^1$. However, when one uses canonical divisors and regular forms, it is indispensable to introduce a certain notion of birational equivalence. With this in mind, proper birational map will be introduced in § 2 and proper birational equivalence between V_1 and V_2 will be taken to mean the existence of a proper birational map between V_1 and V_2. Moreover, logarithmic plurigenera and logarithmic irregularity will be introduced, which are invariants of proper birational equivalence classes. In § 3, we shall raise a couple of conjectures concerning the classification of algebraic varieties, in which *logarithmic Kodaira dimension* will play the most important role. Note that our classification contains that of affine rings. For instance, Theorem 7 is a numerical characterization of $\boldsymbol{C}[x, y, x^{-1}, y^{-1}]$, which is the counterpart of the Enriques criterion on abelian surfaces.

§ 1. **Notation.** We shall work in the category of schemes over $k=\boldsymbol{C}$. Let V be an algebraic variety of dimension n. Then by Nagata we have a completion $\bar{V}$ of V, and by Hironaka we have a non-singular model $\mu: \bar{V}^* \to \bar{V}$, in other words, $\bar{V}^*$ is a complete non-singular algebraic variety and μ a proper birational morphism. Furthermore, we may assume that $D^* = \bar{V}^* - \mu^{-1}(V)$ is a divisor of normal crossing type (i.e., D is a sum of non-singular subvarieties D_j and ΣD_j has only normal crossings). Then $l(m(K(\bar{V}^*)+D^*))$ and $\dim \Gamma(\bar{V}^*, \Omega^1 \log D^*)$ are independent of the choice of $\bar{V}^*$. Hence we write

$$\bar{P}_m(V) = l(m(K(\bar{V}^*)+D^*)),$$
$$\bar{q}(V) = \dim \Gamma(\bar{V}^*, \Omega^1 \log D^*), \text{ and}$$