PAPERS CONTRIBUTED

15. *On the Mutual Reduction of Algebraic Equations.*

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Two algebraic equations $F(x)=\overset{\mu}{\Pi}(x-\alpha_\mu)=0$ and $G(y)=\overset{\nu}{\Pi}(y-\beta_\nu)=0$ of degrees m and n respectively, irreducible in the rationality-domain R, being given, let a polynomial $\varphi(x, y)$ with rational coefficients be so chosen, that the mn values $\gamma_{\mu\nu}=\varphi(\alpha_\mu, \beta_\nu)$ are different from each other. These values are the roots of an equation $H(z)=0$ of degree mn in R and $R(\gamma_{\mu\nu})=R(\alpha_\mu, \beta_\nu)$.

Then we have the following very simple and interesting theorem, which seems to have remained unnoticed.

If $H(z)$ breaks up in e factors $h_i(z)$, irreducible in R, and of degree l_i $(i=1, 2,\cdots\cdots, e)$, and if $f_i(x, \beta)$ is the greatest common divisor of $F(x)$ and $h_i[x, \beta]$, and $g_i(y, \alpha)$ of $G(y)$ and $h_i[\alpha, y]$, then

$$F(x)=f_1(x, \beta)f_2(x, \beta)\cdots\cdots f_e(x, \beta),$$

$$G(y)=g_1(y, \alpha)g_2(y, \alpha)\cdots\cdots g_e(y, \alpha)$$

give the decomposition into irreducible factors of $F(x)$ and $G(y)$ in $R(\beta)$ and $R(\alpha)$ respectively.—$h_i[x, y]$ stands for $h_i\{\varphi(x, y)\}$, α or β for any root of $F(x)=0$ or of $G(y)=0$. If $f_i(x, \beta)$ and $g_i(y, \alpha)$ are of degrees m_i and n_i respectively, then it is known that

$$l_i=m_i n=mn_i, \qquad \frac{m}{n}=\frac{m_i}{n_i}\ (i=1, 2,\cdots\cdots, e).$$

Proof is almost redundant. $\gamma_{\mu\nu}=\varphi(\alpha_\mu\beta_\nu)$ denoting a root of $h_i(z)=0$, $h_i[x, \beta_\nu]$ has with $F(x)$ the greatest common divisor of a degree, say $m'>0$, which, on account of the irreducibility of $G(y)=0$ in R, must be independent of ν, so that the greatest common divisor is $f_i(x, \beta_\nu)$ and $m'=m_i$. The total number of the common roots of $F(x)=0$ and $h_i[x, \beta_\nu]=0$, $\nu=1, 2,\cdots\cdots, n$, being l_i, we have $l_i=m_i n$. If now $h_i[\alpha, \beta]=0$