37. *On Some Properties of Orthogonal Functions.*

By Satoru TAKENAKA.

Shiomi Institute, Osaka.

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Let G be a simply connected domain bounded by an analytic curve C of length l on a *Gaussian* plane ; and consider a set of functions $V_\nu(z)$, $(\nu = 0, 1, 2, \ldots)$ which are regular and analytic for all values of z in C and form a complete system of normalized orthogonal functions on C, that is,

$$\frac{1}{l}\int_c V_\nu(\xi)\overline{V_\mu(\xi)}ds \begin{cases} = 0 & \textit{for } \nu \neq \mu, \\ = 1 & \textit{for } \nu = \mu. \end{cases}$$

Then the series

$$\sum_{\nu=0}^{\infty} V_\nu(z)\overline{V_\nu(a)}, \ (a \text{ in } C)$$

is convergent absolutely and uniformly for all values of z in C and represents a *definite* function $K(z, a)$, which is regular and analytic in C and is defined only by the curve C; and a function $f(z)$, which is regular and analytic in C and is squarely integrable on C can be expressed by the following formula [1]:

$$f(z) = \frac{1}{l}\int_c f(\xi)K(z, \xi)ds, \ (z \text{ in } C).$$

By making use of this formula we can prove the following theorem:

Theorem 1. *If a set of functions* $\{f(z)\}$ *have the properties:*

(i) $f(z)$ *is regular and analytic for all values of* z *in* C,

(ii) $f(a) = 0$, $(a$ in $C)$,

(iii) $\frac{1}{l}\int_c |f(\xi)|^2 ds \leq M, \ (M > 0)$,

then among such functions the unique one which gives the maximum of $|f(x)|$, $(x$ in $C)$ *is given by*

1) S. TAKENAKA, On the orthogonal functions and a new formula of interpolation. This paper will appear in Japanese Journ. of Math. 3 (1926).