148. *On Some Properties of Meromorphic Functions.*

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Consider a class C of meromorphic functions

$$f(z)=\sum_{n=0}^{\infty} b_n z^n \Big/ \sum_{n=0}^{\infty} c_n z^n, \tag{1}$$

where $\sum_{n=0}^{\infty} b_n z^n$ and $\sum_{n=0}^{\infty} c_n z^n$ are integral functions with the following properties:

1) $|b_0|>\varepsilon>0,\ |c_0|>\varepsilon'>0$, and $|b_0-c_0|>\varepsilon''>0$, (2)

2) $|b_n|<L_n$ and $|c_n|<L'_n$ for $n=0, 1, 2,$ (3)

where L_n and L'_n are positive numbers such that $\sum_{n=0}^{\infty} L_n z^n$ and $\sum_{n=0}^{\infty} L'_n z^n$ are also integral functions,

3) of the two sets of inequalities

$$\left.\begin{array}{ll} \text{i)} & 0<l_n<|b_n| \text{ for } n=n_1, n_2, \cdots\cdots\cdots \\ \text{ii)} & 0<l'_{n'}<|c_{n'}| \text{ for } n'=n'_1, n'_2, \cdots\cdots \end{array}\right\} \tag{4}$$

where l_n and $l'_{n'}$ are any positive constants for a given sequence of suffixes $n=n_1, n_2,\cdots\cdots\cdots$ and $n'=n'_1, n'_2\cdots\cdots\cdots$ respectively, at least one is satisfied.

Then we have the following

Theorem: There exists an infinite number of concentric ring-regions $|z|<R_1$ *and* $R_i<|z|<R_{i+1}$ $(i=1, 2,\cdots\cdots)$, R_i *depending only on the class* C, *in which all the functions* (1) *take at least* p *times the value* 1, *or* q *times zero, or have* r *poles.*

Proof.[1] From (2) and (3) it follows that

$$|f^{(n)}(0)|<L''_n \text{ for } n=0, 1, 2,\cdots\cdots \tag{5}$$

where L''_n are finite quantities depending on L_n, L'_n and ε'. First, there

1) A more detailed proof and allied theorems will appear in Proc. Phy-Math. Soc. Japan, Ser. (3), 8 (1926).