15. *Asymmetric Vibrations of Finite Amplitudes.*

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In the former communication[1], asymmetric vibration of small amplitude was discussed, but the treatment of the problem does not show any difference when the amplitude is finite and presents many novel features, which have important bearings on many physical phenomena. Denoting the displacement by ξ, the equation of motion is

$$m\frac{d^2\xi}{dt^2} = -f\xi + g\xi^2 \tag{1}$$

If $g > 0$, then for finite displacement, the restitutive force becomes repulsive when ξ becomes so large that $f < g\xi$, so that the displacement may ultimately become infinitely great. It was formerly assumed that $g > 0$, but the case $g < 0$ can be treated exactly in the same manner.

The first integral of (1) being

$$\frac{m}{2}\left(\frac{d\xi}{dt}\right)^2 = a - \frac{f\xi^2}{2} + \frac{g}{3}\xi^3, \tag{2}$$

we obtain the equation

$$\frac{d\xi}{\sqrt{\xi^3 - \frac{3}{2}\frac{f}{g}\xi^2 + \frac{3a}{g}}} = \frac{d\xi}{\sqrt{(\xi-\alpha)(\xi-\beta)(\xi-\gamma)}} = \frac{d\xi}{\sqrt{R(\xi)}} = \sqrt{\frac{2g}{3m}}\,dt, \tag{3}$$

where α, β, γ are the roots of the cubic under the radical; they are all real when the condition $6ag^2 < f^3$ is satisfied. Suppose that $\alpha > \beta > \gamma$, and put $k^2 = \frac{\beta-\gamma}{\alpha-\gamma}$, $k'^2 = \frac{\alpha-\beta}{\alpha-\gamma}$, then

$$\frac{d\xi}{\sqrt{R(\xi)}} = \frac{2}{\sqrt{\alpha-\gamma}}\frac{dz}{\sqrt{(1-z^2)(1-k^2z^2)}} = \frac{2}{\sqrt{\alpha-\gamma}}du$$

$$= \frac{2i}{\sqrt{\alpha-\gamma}}\frac{dz}{\sqrt{(1-z^2)(1-k'^2z^2)}} = \frac{2i}{\sqrt{\alpha-\gamma}}du'$$

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1) NAGAOKA: Proc. Imp. Acad. 3 (1927) 28.