89. *Fundamental Forms in the Projective Differential Geometry of m-parametric Families of Hypersurfaces of the Second Order in the n-Dimensional Space.*

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1. As I have already reported in another place, it seems to be very natural, to consider hypersurfaces of the second order as space-elements, when we intend to establish the projective differential geometry in the n-dimensional space.[1] Therefore the first problem to deal with is the projective theory of m-parametric families of hypersurfaces of the second order. In this paper I shall determine the fundamental forms in the projective differential geometry of m-parametric families of hypersurfaces of the second order and then add their geometrical meanings.

2. *Notations.* In homogeneous point-coordinates x_λ ($\lambda=0, 1, 2,\ldots \ldots, n$) an m-parametric family of hypersurfaces of the second order is represented by the equation

$$\sum_{\lambda,\mu} \overline{a_{\lambda\mu}}\, x_\lambda x_\mu = 0,$$

where
$$\overline{a_{\lambda\mu}} = \overline{a_{\lambda\mu}}(u^1, u^2, \ldots\ldots, u^n).$$

Then let us take

$$a_{\lambda\mu} = \left\{(n+1)\varDelta\right\}^{-\frac{1}{n+1}} \overline{a_{\lambda\mu}}$$

as its normalized coordinates, where we represent the determinant $|\overline{a_{\lambda\mu}}|$ by $\varDelta$. Now we denote briefly a system of numbers $a_{\lambda\mu}$ by $\mathfrak{a}$, fol-

1) See my previous papers: On the projective differential geometry of plane curves and one-parameter families of conics, these Proceedings, **2**, 307-309, 1926; and Projective differential geometrical properties of the one-parameter families of point-pairs in the one-dimensional space, these Proceedings, **3**, 6-8, 1927. See also my papers: Über die projektive Differentialgeometrie I, II, III, Tôhoku Math. Journal, **28**, 1927.