88. *Analytic Proof of Blaschke's Theorem on the Curve of Constant Breadth with Minimum Area.*

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That the Reuleaux triangle consisting of three circular arcs of radius a is a curve of constant breadth a with minimum area was geometrically proved by Prof. Blaschke in Mathematische Annalen 76,1915. The aim of this note is to prove this theorem analytically.

Take a point on a given curve C of constant breadth a as the origin and a supporting line (Stützgerade) at this point as the initial line. Then the curve C may be represented by the polar-tangential equation of the form $p=p(\theta)$, where $p(0)=p'(0)=0$. As already shown by Prof. Kakeya,[1] the curve of constant breadth a is characterized by the relations

$$\int_0^\pi \rho(\theta)\sin\theta\, d\theta=a,\quad \int_0^\pi \rho(\theta)\cos\theta\, d\theta=0,$$

$$0\leqq\rho(\theta)\leqq a,\qquad \rho(\theta)+\rho(\theta+\pi)=a,$$

where $\rho(\theta)$ denotes the radius of curvature and satisfies

$$\rho(\theta)=p(\theta)+p''(\theta),\quad p(\theta)=\int_0^\theta \rho(\varphi)\sin(\theta-\varphi)d\varphi.$$

The area S of C being equal to $\frac{1}{2}\int_0^{2\pi} p(\theta)\rho(\theta)d\theta$, our problem may be formulated analytically as follows:

$$S=\frac{1}{2}\int_0^{2\pi} d\theta\int_0^\theta \rho(\theta)\rho(\varphi)\sin(\theta-\varphi)\, d\varphi=\text{Minimum},$$

1) Fujiwara-Kakeya, On some problems of maxima and minima for the curve of constant breadth and the in-revolvable curve of the equilateral triangle, Tôhoku Math. Journ., 11 (1917).