133. *The Theory of Two-Dimensional Manifolds in the Projective Space of Four Dimensions.*

By Akitsugu KAWAGUCHI.

Mathematical Institute, Tohoku Imperial University, Sendai.

(Rec. Sept. 21, 1927. Comm. by M. FUJIWARA, M.I.A., Oct. 12, 1927.)

In the theory of two-dimensional manifolds F_2 in R_4 there are generally many facts different from those in R_3. In 1897 K. Kommerell established the theory in the euclidean space,[1] and the theory in the affine space was developed by C. Burstin and W. Mayer in 1927.[2] In this note I will build up this theory in the projective space.[3] The following results can be modified very easily so as to fit also to the affine space.

1. Let us represent F_2 in parametrical form by $\mathfrak{x}=\mathfrak{x}\ (u^1,\ u^2)$, where $\mathfrak{x}$ denotes a system of homogeneous coordinates $x^i\ (i=1, 2, \ldots\ldots, 5)$ of a point. We consider at first a differential form

$$G_2^* \equiv G_{ij} du^i du^j = |\ \mathfrak{x}\ \mathfrak{x}_1\ \mathfrak{x}_2\ d\mathfrak{x}_1\ d\mathfrak{x}_2\ | .$$

which is apparently invariant under the projective transformation group, where

$$\mathfrak{x}_1 = \frac{\partial \mathfrak{x}}{\partial u^1}, \qquad \mathfrak{x}_2 = \frac{\partial \mathfrak{x}}{\partial u^2}.$$

By a transformation of parameters $u^i = u^i\ (\bar{u}^1, \bar{u}^2)$, the forms G_2 and G_{ij} are transformed as follows:

$$G_2^* = \Delta^2 \overline{G}_2^*, \qquad G_{ul} = \Delta^2 \overline{G}_{ij} \frac{\partial \bar{u}^i}{\partial u^u} \frac{\partial \bar{u}^j}{\partial u^l},$$

where
$$\Delta = \frac{\partial(u^1, u^2)}{\partial(\bar{u}^1, \bar{u}^2)}.$$

---

1) K. Kommerell, Die Krümmung der zweidimensionalen Gebilds im ebenen Raume von vier Dimensionen, Dissertation, Tübingen, 1897; Riemannsche Flächen im ebenen Raume von vier Dimensionen, Math. Ann., **60** (1905).

2) C. Burstin and W. Mayer, Über affine Geometrie XVI. Die Geometrie zweifach ausgedehnter Mannigfaltigkeiten F_2 im affinen R_4, Math. Zeits., **26** (1927), p. 371-407.

3) G. Fubini developed also this theory in another method. G. Fubini—E. Čech, Geometria proiettiva differenziale, Bologna, 1927, p. 631-649.