PAPERS COMMUNICATED

77. *Differential Geometry of Conics in the Projective Space of Three Dimensions.*

I. Fundamental Theorem in the Theory of a One-parameter Family of Conics.

By Akitsugu KAWAGUCHI.

(Rec. May 28, 1928. Comm. by M. FUJIWARA, M.I.A., June 12, 1928.)

The elements in the geometry are, in general, arbitrary, so by adopting different elements we can find various new results, which are important and interesting from geometrical points of view. In the differential geometry the theories of lines and of circles (line-geometry and circle-geometry) have been already developed by many authors, but the theory of conics has, up to the present, never been discussed except in some papers dealing with algebraic geometry[1], and remains unsolved after having been proposed by F. Klein in his Erlanger Programm. I will now, in the present note and also in the papers which will appear in the near future, discuss the differential geometry of conics in the projective space of three dimensions R_3. The theory of quadratic cones is completely dual to the present theory in the projective space.

1. *Coordinates of a conics in R_3.* Let the homogeneous coordinates of a plane L in R_3 be l_i $(i=1, 2, 3, 4)$ and consider the homogeneous coordinates of a point on the plane L: x^α $(\alpha=1, 2, 3)$. In that case, we take, as the triangle of reference on the plane L, the triangle formed by three diagonals of the complete quadrilateral, which is the section of the tetrahedron of reference T in R_3 by the plane L. And let the homogeneous point-coordinates in R_3 be y^i, where the tetrahedron of reference is also T. When we represent the coordinates of a point x^α on the plane L by the coordinates in space y^i and inversely the coordinates y^i of a point in space by the coordinates on the plane passing through that point, we get

$$(1)\qquad \begin{cases} \rho y^1=l_2l_3l_4\,(x^1+x^2+x^3), & \rho y^2=l_1l_3l_4\,(-x^1-x^2+x^3), \\ \rho y^3=l_1l_2l_4\,(-x^1+x^2-x^3), & \rho y^4=l_1l_2l_3\,(x^1-x^2-x^3)\,; \end{cases}$$

$$(2)\qquad \begin{cases} \sigma x^1=l_1y^1-l_2y^2-l_3y^3+l_4y^4, & \sigma x^2=l_1y^1-l_2y^2+l_3y^3-l_4y^4, \\ \sigma x^3=l_1y^1+l_2y^2-l_3y^3-l_4y^4, & \end{cases}$$

where

$$(3)\qquad l_iy^i \doteq 0 \quad \text{and} \quad \rho\sigma=4.$$

---

1) See R. A. Johnson, The conic as a space element, Trans. of American Math. Soc., **15** (1914); and C.G.F. James, Analytic representation of congruences of conics, Proc. Camb. Phil. Soc. **21** (1922–1923).