97 On Sufficient Conditions for the Uniqueness of the Solution of $\frac{dy}{dx}=f(x, y)$.

By Tatsujirô SHIMIZU.
Mathematical Institute, Tokyo Imperial University.

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We consider the differential equation

$$\frac{dy}{dx}=f(x, y)\ , \tag{1}$$

where $f(x, y)$ is a continuos function of x and y in the domain D $(0\leqq x\leqq a,\ |y|\leqq b)$. The equation (1) has always at least an integral curve which passes through $x=0,\ y=0$. For the uniqueness of the integral curve of (1) many sufficient conditions are known. Besides the well-known Lipschitz's condition $|f(x, y_1)-f(x, y_2)|<K|y_1-y_2|$, a sufficient condition

$$|f(x, y_1)-f(x, y_2)|<K|y_1-y_2|\log\frac{1}{|y_1-y_2|} \tag{2}$$

or more generally

$$|f(x, y_1)-f(x, y_2)|<\varphi(|y_1-y_2|), \text{ where } \lim_{y=0}\int_\delta^y\frac{dy}{\varphi(y)}=-\infty, \tag{3}$$

was given by Osgood,[1] and another condition

$$|f(x, y_1)-f(x, y_2)|<k\frac{|y_1-y_2|}{x},\quad 0\leqq k<1, \tag{4}$$

by Rosenblatt.[2]

Recently Nagumo[3] without knowing Rosenblatt's condition (4) has discovered a more general condition

$$|f(x, y_1)-f(x, y_2)|<\frac{|y_1-y_2|}{x}\ . \tag{5}$$

Nagumo[4] and Perron[5] have extended the condition (5) to

$$|f(x, y_1)-f(x, y_2)|\leqq\frac{|y_1-y_2|}{x}\ . \tag{6}$$

Further Perron[6] has shown by simple examples that

1) Osgood, Monatshefte für Math. und Phys. **9** (1898) 331.
2) Rosenblatt, Arkiv för Mat. Astr. och Fys. **5** (1909) 2, 1.
3) Nagumo, Japanese Jour. of Math. **3** (1926) 107.
4) Nagumo, Japanese Jour. of Math. **4** (1927) 307.
5) Perron, Math. Zeitschr. **28** (1928) 216.
6) Perron. ibid.