157. *On the Class of Functions with Absolutely Convergent Fourier Series.*

By Kien-Kwong CHEN.
Mathematical Institute, Tohoku Imperial University, Sendai.

(Comm. by M. FUJIWARA, M.I.A., Nov. 2, 1928.)

1. Let

$$\frac{1}{2}a_0+\sum_{n=1}^{\infty}(a_n\cos nx+b_n\sin nx) \tag{1}$$

be the Fourier series of a periodic summable function $f(x)$ with the period 2π. As regards the absolute convergence of the series (1), Zygmund[1] has given a sufficient condition in the form that the function $f(x)$ is of limited variation and satisfies Lipschitz's condition of the positive order.

In this note, we determine the class of all the functions whose Fourier series converge absolutely.

A periodic function $f(x)$ is said to be Young's continuous function, if there exist two periodic square-summable functions $f_1(x)$, $f_2(x)$, satisfying the relation

$$f(x)=\frac{1}{\pi}\int_{-\pi}^{\pi}f_1(\xi)f_2(\xi+x)d\xi\,,$$

here and afterwards the period being taken to be 2π. The functions of such a type were first considered by Young[2]. Now we will prove the following theorem:

The necessary and sufficient condition for the absolute convergence of a trigonometrical series in the whole interval[3]*, is that the series is a Fourier series of a Young's continuous function.*

2. First we prove the necessity of the condition. Assuming the absolute convergence of the series

$$\frac{1}{2}a_0+\sum_{n=1}^{\infty}(a_n\cos nx+b_n\sin nx)\,, \tag{1}$$

1) A. Zygmund, Remarque sur la convergence absolue des séries de Fourier, The Journal of the London Math. Soc., **3** (1928), 194-196.

2) W.H. Young, On a class of parametric integrals etc., Proc. Roy. Soc. (A), **85** (1911), 401-414.

3) N. Lusin proved that if a trigonometrical serie is absolutely convergent at a set of positive measure, it converges everywhere absolutely; see Comptes Rendus, **155** (1912), 580.