PAPERS COMMUNICATED

19. *A Generalization of Tauber's Theorem.*

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1. It was proved by Prof. Tauber[1)] that:

If $na_n \to 0$ *as* $n \to \infty$, *and if*

$$\lim_{x\to 1-0} \sum_{n=1}^{\infty} a_n x^n = A ,$$

then

$$\sum a_n = A .$$

The condition $na_n \to 0$ was replaced by the broader condition $a_n = O\left(\frac{1}{n}\right)$ by Prof. Littlewood[2)], and Professors Hardy and Littlewood[3)] replaced it again by $na_n > -K$. Finally Dr. R. Schmidt[4)] proved that it is sufficient to assume

$$\varliminf_{m,\, n\to\infty} (s_m - s_n) \geqq 0 ,$$

when $m > n$ and $m/n \to 1$.

On the other hand Prof. Littlewood[5)] proved that:

Suppose that

$$0 < \lambda_{n-1} < \lambda_n , \qquad \lambda_n \to \infty ,$$

(1) $$\frac{\lambda_n - \lambda_{n-1}}{\lambda_n} \to 0 ;$$

and further that

$$a_n = O\left(\frac{\lambda_n - \lambda_{n-1}}{\lambda_n}\right) ;$$

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1) Tauber: Monatshefte für Math. u. Physik, **8** (1897).
2) Littlewood: Proc. London Math. Soc. (2) **9** (1910).
3) Hardy-Littlewood: ibid. (2) **13** (1913).
4) R. Schmidt: Math. Zeits. **22** (1925). The direct proof of this theorem was given by Dr. Vijayaraghavan (Journ. London Math. Soc **1** (1916)).
5) Littlewood: loc. cit.