# PAPERS COMMUNICATED

## 129. *On the Order of the Absolute Value of a Linear Form (Fifth Report).*

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**1.** Let us consider the function $\varphi_{\alpha,\beta}(t)$, which is the minimum absolute value of $t(\alpha x-y-\beta)$ for the integral values of $x$ and $y$, where $|x|<t$. In the former reports I treated mainly two problems: the problem of finding the inferior limit of this function, which may be considered as an extension of a problem solved by Minkowski, and that of finding the superior limit of this function, which was proposed by Hardy and Littlewood. The methods, which I used to solve these two problems were different from each other, although they stand on the same idea. This fact is inconvenient for the further discussion in each problem. In this note I wish to establish a new algorithm, which gives us the relation between the two former algorithms. As the first application of this algorithm I will study the nature of the approximate polygon of the first algorithm more precisely and find the best approximation with regard to the nature of the approximate polygon.

**2.** *The new algorithm.* We take on the xy-plane a system of lattice points, corresponding to the integral values of $x$ and $y$, and the straight line $L: \alpha x-y-\beta=0$ and $Y: x=0$, whose intersection is supposed to be $M$. First we construct a parallelogramm containing $M$ in it, whose sides are parallel to $L$ and $Y$ and which contains no lattice point. Then we translate each side until a lattice point comes on it. We construct all the parallelogramms of such character and call them the approximate parallelogramms. The lattice points on the sides of approximate parallelogramms are called the approximate points. We choose a series of the groups of approximate points $(A_1, B_1, C_1, D_1)$, $(A_2, B_2, C_2, D_2)$, ......, $(A_n, B_n, C_n, D_n)$, ...... and by an affine transformation transform the points $A_n$, $B_n$, $C_n$, $D_n$ into (0,1), (0,0), $(-1,0)$ $(-1,1)$, and let the new positions of $L$ and $Y$ be $L_n: \alpha_n x-y-\beta_n=0$ and $Y_n: \alpha_n' x+y+\beta_n'=0$. By the similar method as in the former reports we can find the sequence $(a_n, b_n, c_n, \tau_n)$, which satisfies the following relations: