PAPERS COMMUNICATED

90. *On the System of Linear Inequalities.*

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In my former paper, On the system of linear inequalities and linear integral inequality, this Proceedings, 4 (1928), I have shown that the condition for the existence of the solutions of a system of linear inequalities is equivalent to the condition for the existence of *positive* solutions of a certain system of linear equations, and have given the said necessary and sufficient condition in the following form.

In order that the system of linear equations

$$\sum_{k=1}^{m} a_{ik}x_k=0\,, \qquad (i=1,\ 2,\ \ldots\ldots,\ n)$$

has a system of positive solutions $x_1, x_2, \ldots\ldots, x_n > 0$, it is necessary and sufficient that the origin must lie within the smallest convex polyhedron S containing the following m points in the n-dimensional space:

$$P_i(a_{1i},\ a_{2i},\ \ldots\ldots,\ a_{ni})\,, \qquad (i=1,\ 2,\ \ldots\ldots,\ m).$$

Recently, in a paper, Huber: Eine Erweiterung der Dirichletschen Methode des Diskontinuitätsfaktors und ihre Anwendung auf eine Aufgabe des Wahrscheinlichkeitsrechnung, Monatshefte für Mathematik und Physik 37 (1930), the following theorem and its proof due to Furtwängler are published:

If at least one of r linear forms

$$L_i(x)=\sum_{k=1}^{n} a_{ik}x_k\,, \qquad (i=1,\ 2,\ \ldots\ldots,\ r) \tag{1}$$

for any system of non-negative $x_1, x_2, \ldots\ldots, x_n \geqq 0$ becomes positive, then there exists a system of positive numbers $p_1, p_2, \ldots\ldots, p_r > 0$ such that

$$M_i(p)=\sum_{k=1}^{r} a_{ki}p_k > 0\,, \qquad (i=1,\ 2,\ \ldots\ldots,\ n). \tag{2}$$

The existence of a system of positive numbers $p_1, p_2, \ldots\ldots, p_r$ such that the relation (2) holds good is nothing but the existence of the *positive* solutions $x_1, x_2, \ldots\ldots, x_{n+r}$ for the system of linear equations:

$$\sum_{k=1}^{r} a_{ki}x_k=x_{r+i}\,, \qquad (i=1,\ 2,\ \ldots\ldots,\ n).$$