40. On the Distribution of Zero Points of the Derivatives of an Integral Transcendental Function of Order $\rho \leqq 1$.

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(Comm. by M. FUJIWARA, M.I.A., April 13, 1931.)

1. Recently I have proved the following theorem which is a modified form of a theorem enunciated by Mr. Takahashi:[1]

THEOREM I.[2] *Let $\{g_n(z)\}$ be a sequence of functions satisfying the following conditions:*

(i) *$g_n(z)$ is regular and analytic for $|z| \leqq R$;*

(ii) *$g_n(z) = z^n\{1+h_n(z)\}$, where $h_n(z)$ is regular and analytic for $|z| \leqq R$ and vanishes at the origin;*

(iii) *there exists a finite constant λ for which*

$$\overline{\lim_{n=\infty}} |h_n(z)| \leqq \lambda \qquad \text{for} \qquad |z| \leqq R .$$

Then any function $f(z)$ regular and analytic for $|z| \leqq r$ can be expanded in one and only one way into the series of the form

$$f(z) = \sum_{n=0}^{\infty} c_n g_n(z) ,$$

which converges absolutely and uniformly for

$$|z| \leqq r_0 < \min\left(r, \frac{R}{1+\lambda}\right).$$

2. Let us consider a set $\{\alpha_n\}$ of points such that

$$\overline{\lim_{n=\infty}} |\alpha_n| = L$$

and put

$$g_n(z) = z^n e^{\bar{\alpha}_n z} = z^n\{1+h_n(z)\}, \quad (|z| \leqq R, \ n=0, 1, 2, \ldots\ldots) .$$

Since $g_n(z)$ is regular and analytic for any finite value of R, and moreover

$$|h_n(z)| \leqq e^{|\alpha_n| R} - 1 \qquad \text{for} \qquad |z| \leqq R ,$$

$$\overline{\lim_{n=\infty}} |h_n(z)| = e^{LR} - 1 (= \lambda \text{ say}) \qquad \text{for} \qquad |z| \leqq R ,$$

1) S. Takahashi: A remark on Mr. Widder's theorem, Tohoku Math. Journal, **33** (1930), 48.

2) The proof of this theorem will be given in my paper "On the expansion of analytic functions in a series of analytic functions and its applications etc." which will appear in Proc. Phys.-Math. Soc. of Japan.