90. Analytic Proof of Blaschke's Theorem on the Curve of Constant Breadth, II.

By Matsusaburo FUJIWARA, M.I.A.
Mathematical Institute, Tohoku Imperial University.

(Comm. Oct. 12, 1931.)

In the former paper with the same title, this Proceedings **3**, 1927, I have given an analytic proof of Blaschke's theorem:

The Reuleaux triangle consisting of three circular arcs of radius a is a curve of constant breadth a with minimum area.

There I have only sketched the main line of proof and left untouched the proof of the fact, that we can determine A and B such that

$$L(\theta)+a\leqq 0 \qquad \text{for} \qquad 0\leqq\theta<\frac{\pi}{3},$$

$$L(\theta)+a\cos\left(\frac{\pi}{3}-\theta\right)\geqq 0 \qquad \text{for} \qquad \frac{\pi}{3}\leqq\theta<\frac{2\pi}{3},$$

$$L(\theta)+a(1+\cos\theta)\leqq 0 \qquad \text{for} \qquad \frac{2\pi}{3}\leqq\theta<\pi,$$

where $$L(\theta)=\int_0^{\theta}\rho(\varphi)\sin(\theta-\varphi)d\varphi+A\cos\theta+B\sin\theta-a.$$

When I recently informed my proof to Mr. Morimoto, he remarked me a slight error in it. So I will give here the corrected proof in detail.

Determining A and B such that

$$0=L(\theta)+a=L(\theta)+a\cos\left(\frac{\pi}{3}-\theta\right) \qquad \text{for} \qquad \theta=\frac{\pi}{3},$$

$$0=L(\theta)+a\cos\left(\frac{\pi}{3}-\theta\right)=L(\theta)+a(1+\cos\theta) \qquad \text{for} \qquad \theta=\frac{2\pi}{3},$$

and putting these values in $L(\theta)$, we get

$$\begin{aligned}L(\theta)=&-a-\frac{a}{\sqrt{3}}\sin\left(\frac{\pi}{3}-\theta\right)+\int_0^{\theta}\rho(\varphi)\sin(\theta-\varphi)d\varphi\\&+\frac{2}{\sqrt{3}}\sin\left(\frac{\pi}{3}-\theta\right)\int_0^{\frac{2\pi}{3}}\rho(\varphi)\sin\left(\frac{2\pi}{3}-\varphi\right)d\varphi\\&-\frac{2}{\sqrt{3}}\sin\left(\frac{2\pi}{3}-\theta\right)\int_0^{\frac{\pi}{3}}\rho(\varphi)\sin\left(\frac{\pi}{3}-\varphi\right)d\varphi.\end{aligned}$$