PAPERS COMMUNICATED

17. On the Expansion of an Integral Transcendental Function of the First Order in Generalized Taylor's Series.

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1. In my previous paper[1] I have proved the following theorem:

THEOREM A. *Let* $\{a_n\}$ *be a set of points such that*

$$\overline{\lim_{n=0}}|a_n|=L<\infty$$

Then any function $\phi(z)$, *regular and analytic for* $|z|<r$, *can be expanded in one and only one way into the series of the form*

$$\phi(z)=\sum_{n=0}^{\infty}c_n z^n e^{\bar{a}_n z} \tag{1. 1}$$

which converges absolutely and uniformly for $|z|\leq r_0<\min\left(r, \frac{1}{eL}\right)$.

Let us define a sequence $\{p_n(z)\}$ of polynomials by

$$p_0(z)=1\,,\quad p_n(z)=\int_{a_0}^{z}\int_{a_1}^{t_1}\cdots\cdots\int_{a_{n-1}}^{t_{n-1}}dt_n dt_{n-1}\cdots\cdots dt_1\,,\quad (n\geqq 1) \tag{1. 2}$$

which satisfy the equalities:

$$p_n^{(\nu)}(a_\nu)=\begin{cases}0 & \text{for } \nu\neq n\,,\\ 1 & \text{for } \nu=n\,,\end{cases} \tag{1. 3}$$

and put

$$p_n(z)=\sum_{\nu=0}^{n}\frac{k_\nu^{(n)}}{\nu\,!}z^\nu\,,\qquad (n=0,\,1,\,2,\,\ldots\ldots)\,,$$

and define a sequence $\{\pi_n(z)\}$ of polynomials by

$$\pi_n(z)=\sum_{\nu=0}^{n}k_\nu^{(n)}z^\nu\,,\qquad (n=0,\,1,\,2,\,\ldots\ldots)\,.$$

Then it can easily be shown that

$$\begin{cases}p_n(z)=\dfrac{1}{2\pi}\displaystyle\int_{|\zeta|=1}\pi_n(\zeta)e^{z\bar{\zeta}}\,|d\zeta|\,,\\ p_n^{(\nu)}(a_\nu)=\dfrac{1}{2\pi}\displaystyle\int_{|\zeta|=1}\pi_n(\zeta)\bar{\zeta}^\nu e^{a_\nu\bar{\zeta}}\,|d\zeta|\,,\qquad (n,\,\nu=0,\,1,\,\ldots\ldots)\,,\end{cases} \tag{1. 4}$$

1) S. Takenaka: On the distribution of zero points of the derivatives of an integral transcendental function of order $\rho\leq 1$, Proc. **7** (1931), 134.