62. A Generalization of Ostrowski's Theorem on "Overconvergence" of Power Series.

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In my previous paper,[1] I have proved that a function $f(z)$, regular and analytic for $|z|<r (r>1)$, can be expanded into the series of the form

$$(1) \qquad f(z)=\sum_{n=0}^{\infty} c_n z^n e^{\bar{\alpha}_n z}$$

which converges absolutely and uniformly for $|z|\leqq 1$ provided that

$$\overline{\lim_{n=\infty}}|\alpha_n|=L<\frac{1}{e}.$$

Let $\{\pi_n(z)\}$ be a sequence of polynomials defined by

$$p_n(z)=\frac{1}{2\pi}\int_{|\zeta|=1}\pi_n(\zeta)e^{z\bar{\zeta}}|d\zeta|, \qquad (n=0, 1, 2, \ldots\ldots)$$

where $\quad p_0(z)=1, \quad p_n(z)=\int_{\alpha_0}^{z}\int_{\alpha_1}^{t_1}\ldots\ldots\int_{\alpha_{n-1}}^{t_{n-1}}dt_n dt_{n-1}\ldots\ldots dt_1, \qquad (n\geqq 1).$

Since $\{\pi_n(z)\}$ and $\{z^n e^{\bar{\alpha}_n z}\}$ are each other biorthogonal[2] on $|z|=1$, we have, from (1),

$$\frac{1}{1-\bar{x}z}=\sum_{n=0}^{\infty}\overline{\pi_n(x)}\,z^n e^{\bar{\alpha}_n z}, \qquad (|x|<1, \ |z|<\frac{1}{|x|})$$

or

$$(2) \qquad \frac{1}{\zeta-x}=\sum_{n=0}^{\infty}\pi_n(x)\frac{1}{\zeta^{n+1}}e^{\frac{\alpha n}{\zeta}}, \qquad (|x|<1, \ |\zeta|>|x|)$$

the series on the right hand side of (2) being convergent absolutely and uniformly for $|\zeta|\geqq r'>|x|$.

Now let $f(z)$ be a function, regular and analytic for $|z|<1$, with at least one singular point on $|z|=1$. Then the function defined by

$$F(z)=\frac{1}{2\pi i}\int_{|\zeta|<1}f(\zeta)\frac{1}{\zeta}e^{\frac{z}{\zeta}}d\zeta$$

may easily be shown to be an integral transcendental function of type 1 and of the first order, and this can be uniquely determined if

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1) S. Takenaka: On the expansion of an integral transcendental function of the first order in generalized Taylor's series, Proc., 8 (1932), 59.

2) See Takenaka loc. cit.