61. On the Relation between $M(r)$ and the Coefficients of a Power Series.

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The relations between the maximum modulus $M(r)=\operatorname*{Max}_{|z|=r}|f(z)|$ of a power series

$$f(z)=a_0+a_1z+a_2z^2+\cdots\cdots$$

and the order of $|a_n|$ are investigated by many authors, in the case of integral transcendental functions, and some analogous results are obtained in the case of a power series with the convergence radius 1. Dr. Beuermann[1] has recently treated the latter case and given the following result.

If we denote

$$\limsup_{r\to 1-0}\frac{\log\log M(r)}{\log\dfrac{1}{1-r}}=\mu\,,\qquad \limsup_{n\to\infty}\frac{\log\log|a_n|}{\log n}=\sigma\qquad(0<\sigma<1),$$

then there exists the relation

$$\mu=\sigma/(1-\sigma)\,.$$

I will here add the following remark.

Theorem. Let

$$\limsup_{r\to 1-0}\frac{\log M(r)}{(1-r)^{-\mu}}=\kappa\,,\qquad \limsup_{n\to\infty}\frac{\log|a_n|}{n^\alpha}=\beta\,,$$

$$(\mu>0,\ \ \kappa,\ \beta\ \text{finite}\neq 0,\ \ 0<\alpha<1),$$

then $$\mu=\alpha/(1-\alpha)\,,\qquad \kappa=\beta^{\frac{1}{1-\alpha}}(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}}\,.$$

The method is not essentially new; it is only an application of Laplace's method concerning the functions of large numbers.

Let $$\limsup_{n\to\infty}\frac{\log|a_n|}{n^\alpha}=\beta\qquad(0<\alpha<1)\tag{1}$$

be finite. Then for any $\varepsilon>0$, we can determine m such that

$$\frac{\log|a_n|}{n^\alpha}<\beta+\varepsilon=\gamma\,,$$

i.e. $$|a_n|<e^{\gamma n^\alpha}\qquad\text{for}\qquad n\geqq m\,,$$

1) Beuermann, Math. Zeits. **33** (1931).