PAPERS COMMUNICATED

77. *On the Starshaped Mapping by an Analytic Function.*

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(Comm. by T. YOSIE, M.I.A., July 12, 1932.)

1. Our object is to prove the following

Theorem. *Let*

$$f(z)=z+a_2z^2+\cdots\cdots+a_nz^n+\cdots\cdots$$

be regular for $|z|<R$ *and* $|f'(z)|<M$ *for* $|z|<R$. *Then the circle* $|z|<\dfrac{R}{M}$ *is mapped on a starshaped domain with respect to the origin by* $f(z)$ *and also by all its polynomial sections*

$$f_n(z)=z+a_2z^2+\cdots\cdots+a_nz^n \qquad (n=1,\ 2,\ \ldots\ldots).$$

Moreover the limiting case is attained by the function

$$f(z)=MR\Big(\frac{M}{R}z+(M^2-1)\log\Big(1-\frac{z}{MR}\Big)\Big).$$

This is a more precise form of a theorem due to S. Takahashi.[1]

2. First we will enunciate a lemma, which is of some interest.

Lemma. *Let* $f(z)=z+a_2z^2+\cdots\cdots+a_nz^n+\cdots\cdots$ *be regular in the unit circle. If* $\sum_2^\infty n|a_n|r^{n-1}<1$, $0<r<1$, *the circle* $|z|\leqq r$ *is mapped on a starshaped domain with respect to the origin by* $f(z)$ *and also by every section* $f_n(z)$.

It is known that $f(z)$ and every section $f_n(z)$ are univalent (schlicht) for $|z|\leqq r$.[2] Therefore $z\dfrac{f'(z)}{f(z)}$ and $z\dfrac{f_n'(z)}{f_n(z)}$ $(n=1,\ 2,\ \ldots\ldots)$ are regular for $|z|\leqq r$. For the proof it is sufficient to show that

$$R\Big[z\frac{f'(z)}{f(z)}\Big]>0 \quad \text{and} \quad R\Big[z\frac{f_n'(z)}{f_n(z)}\Big]>0^{3)} \quad \text{for} \quad |z|=r.$$

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1) S. Takahashi: Tôhoku Math. Journ., **33** (1930), p. 55–60. T. Tannaka: Tôhoku Math. Journ., **35** (1932), p. 43–46. S. Kakeya: Sci. Rep. of Tokyo Bunrika Daigaku, Sect. A, **1** (1932), p. 238–240.

2) T. Itihara: Jap. Journ. of Math., Vol. **6** (1929). See p. 183–184.

3) $R[\zeta]$ denotes the real part of ζ.