16. A New Concept of Integrals, II.[1)]

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7. Let $f(x)$ be almost everywhere finite in (a, b). $M^*(x)$ is called a *major* function* of $f(x)$ in (a, b), if it satisfies the following conditions:

1°. $M^*(x)$ is (τ)-approximately continuous in the closed interval $[a, b]$, $(\tau > \frac{1}{2})$.

2°. $M^*(a) = 0$.

3°. (a, b) is covered by a system of enumerable perfect sets $\{P_i\}$, except an enumerable set at most, such that

3°. 1. $\underset{\tau}{A}\underline{D}M_i(x) > -\infty$

with the possible exception of an enumerable set in P_i,

3°. 2. $\underset{\tau}{A}\underline{D}M_i(x) \geqq f(x)$

with the possible exception of an enumerable set in P_i, where $M_i(x)$ is defined such that

$$M_i(x) = M^*(x), \text{ for } x \text{ in } P_i \text{ and for } x = a,\ x = b,$$

and $M_i(x)$ is linear in the contiguous intervals of P_i.

4°. For any perfect subset Q_i of P_i, $N_i(x)$, defined as $M_i(x)$, taking Q_i instead of P_i, has the corresponding properties of $M_i(x)$.

Similarly, a *minor* function* $m^*(x)$ is defined. $M^*(x)$ and $m^*(x)$ are called the associated* functions of $f(x)$ in (a, b).

Theorem 21. If $f(x)$ is defined in (a, b), and $M^*(x)$ and $m^*(x)$ are the associated* functions of $f(x)$, then $M^*(x) - m^*(x)$ is a positive non-decreasing function. In particular,

$$M^*(b) \geqq m^*(b).$$

Suppose that $f(x)$ is defined and is almost everywhere finite in (a, b), and the associated* functions $M^*(x)$ and $m^*(x)$ of $f(x)$ exist.

If we put

$$I_1^*(b) = \text{lower bound of all } M^*(b),$$

and

$$I_2^*(b) = \text{upper bound of all } m^*(b),$$

then they are finite and

1) In the first paper (this volume, No. 10, pp. 570–574), I have to correct the following points: 1°. In Theorem 2 and 3, read $\tau > \frac{1}{2}$ for $\tau > 0$; 2°. To the last of Theorem 6, add $(\tau > \frac{1}{2})$.