PAPERS COMMUNICATED

13. *The Foundation of the Theory of Displacements, II.*

(*Application to the functional manifolds.*)

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(Comm. by M. FUJIWARA, M.I.A., Feb. 12, 1934.)

As applications of the general theory set out in my previous paper[1] I shall take here the functional space manifolds and in the next paper the manifolds of matrices. We may regard the former as of infinitely many dimensions and the latter of indeterminate dimensions.

1. Let the underlying manifold M and the associated manifolds $M^{(i)}$, $\overline{M}$ be all functional manifolds, which consist of summable real functions of a real variable.[2] In this case any element a^* of M^* is a system of functions $a(t)$, $a^{(i)}(t)$, which are elements of M, $M^{(i)}$ respectively. It is natural to take such correspondence between any manifolds $\overline{M}$ as the underlying isomorphism that corresponding functions have same values for all same values of their variables. By the term neighbourhood of a function a^t [3] we understand a totality of functions ξ^t such that $|\xi^t - a^t| < \varepsilon$, ε being a positive number. Consider an element $\bar{a}^t(a^*)$ in $\overline{M}_{a*}$ determined uniquely for every a^* and continuous with respect to a^*, then the covariant change of a function $\bar{a}^t(a^*)$ due to a displacement D_{a*b*} can be represented by

$$\nabla \bar{a}^t = \triangle \bar{a}^t + \Gamma^t(\bar{a}_{a*}, \bar{a}_{b*}, D_{a*b*}), \tag{1}$$

or by notation of differential $\delta \bar{a}^t$ for b^* approaching a^* such that $D_{a*b*} \to 1$

$$\nabla \bar{a}^t(a^*) = \delta \bar{a}^t(a^*) + \Gamma^t(\bar{a}_{a*}, a^*, D_{a*, a*+\delta a*}). \tag{2}$$

2. Now we take the following four postulates.

The first Postulate: *The displacement is linear.*

$$\nabla(\bar{a}^t(a^*) + \bar{b}^t(a^*)) = \nabla \bar{a}^t(a^*) + \nabla \bar{b}^t(a^*), \tag{3}$$

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1) A. Kawaguchi: The foundation of the theory of displacements, Proc. **9** (1933), 351–354 (cit. with F.D.I.).

2) For real or complex functions of several real or complex variables we can also establish a similar theory by some modification.

3) We use the notation a^t for a function $a(t)$ of a real variable t as a matter of convenience.