39. A New Proof of the Andersen's Theorem.

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1. Let
$$\sum_{n=0}^{\infty} a_n \tag{1}$$
be the given series. We put
$$A_n^{(m)}=\binom{m+n}{n},$$
$$S_n^{(r)}=\sum_{\nu=0}^{n} A_{n-\nu}^{(r-1)} s_\nu ,$$
where $s_\nu=a_0+a_1+\cdots\cdots+a_\nu$.

If the limit of
$$\frac{S_n^{(r)}}{A_n^{(r)}} \tag{2}$$
exists and $=s$, then (1) is said to be (C, r)-summable to sum s, and we write $\sum_{n=0}^{\infty} a_n=s\ (C, r)$. If (2) is bounded, then (1) is said to be (C, r)-bounded, and we write $\sum_{n=0}^{\infty} a_n=o(1)\ (C, r)$.

The object of this paper is to prove the following theorems.

Theorem 1. Let $\sigma>\rho>-1$. *If*
$$\sum_{n=0}^{\infty} a_n=O\ (1)(C, \rho)$$
and
$$\sum_{n=0}^{\infty} a_n=s\ (C, \sigma)\,,$$
then $\sum_{n=0}^{\infty} a_n=s\ (C, \tau)$ *for any* $\tau>\rho$.

Theorem 2. Let $\sigma>\rho>-1$. *If*
$$|S_n^{(\rho)}|<A_n^{(\rho)}$$
and
$$|S_n^{(\sigma)}|<A_n^{(\sigma)},$$
then
$$|S_n^{(\tau)}|<\left(2+\frac{\Gamma(\tau-\rho+1)\Gamma(\sigma-\tau+1)}{\Gamma(\sigma-\rho+1)}+o(1)\right)A_n^{(\tau)} \tag{3}$$
for any $\tau>\rho$.

These theorems are due to Andersen.[1] The constant in (3) seems to be new.

1) Andersen: Studier over Cesàro Summabilitetsmetode, 1921. Cf. Zygmund, Math. Zeits., **25** (1926).