

85. Remarks to Some Theorems of Burnside.

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(Comm. by M. FUJIWARA, M.I.A., June 12, 1934.)

Burnside proved the following two theorems concerning the theory of representation of finite order in his Theory of Groups of Finite Order.¹⁾

Theorem 1. If Γ be a representation of G as a group of linear substitutions, and if G is simply isomorphic with Γ , and when the process of compounding Γ with itself is carried far enough, then, every irreducible representation of G will arise.

Theorem 2. If $s(< r)$ of the irreducible representations of G , viz. $\Gamma_1, \Gamma_2, \dots, \Gamma_s$ combine among themselves by composition, then G has a self-conjugate subgroup H , each of whose operation is represented by the identical substitution in these s representations of G and in no others.

To prove these theorems Burnside used the convergence of a power series. In his case the representations of the group are done in the domain of complex numbers. If we take an abstract domain in place of the domain of complex numbers, we must modify slightly his proof. The object of this paper is to give purely algebraic proof to these theorems. In the course of these proofs it is convinced that we can express Theorem 1 more precisely, and further we will see that these two theorems hold good also, even when the group is represented by collineation groups. In the following we will adopt the notations in Prof. Burnside's work.

Proof. If we denote the reduced form of the Kronecker's product Γ^n by the formula

$$\Gamma^n = \sum_i \gamma_{ni} \Gamma_i,$$

then

$$(\psi_p)^n = \sum_p h_p \chi_p^i (\psi'_p)^n,$$

from which we obtain

$$(A) \quad N\gamma_{ni} = \sum_i h_p \chi_p^i (\psi'_p)^n$$

by using the formula

1) p. 298.