# PAPERS COMMUNICATED

## 83. *On the Convergence Factor of Fourier-Lebesgue Series.*

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1. Let $f(t)$ be a summable periodic function with period $2\pi$, and let

$$f(t) \sim \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nt + b_n \sin nt),$$

$$\phi(t) = \frac{1}{2}\{f(x+t) + f(x-t) - 2f(x)\},$$

and

$$\phi_\alpha(t) = \frac{1}{\Gamma(\alpha)} \int_0^t (t-u)^{\alpha-1} \phi(u) du.$$

Then we have

Theorem A.[1] *If* $\alpha > 0$ *and*

$$\phi_\alpha(t) = o(t^\alpha), \tag{1.1}$$

*then the series*

$$\sum_{n=1}^{\infty} \frac{a_n \cos nt + b_n \sin nt}{n^{\frac{\alpha}{\alpha+1}}}$$

*is convergent for* $t=x$.

A summable function $f(t)$ is said to belong to $L_p$, or simply, $f(t) \in L_p$ provided that its $p$-th power $|f(t)|^p$ is summable in $(-\pi, \pi)$. The object of this paper is to prove some related theorems as Theorem A.

Theorem 1. *If* $f(t) \in L_p$ $(p > 1)$, *and*

$$\int_0^t \phi(t) dt = o(t), \tag{1.2}$$

*then the series*

$$\sum_{n=1}^{\infty} \frac{a_n \cos nt + b_n \sin nt}{n^{\frac{1}{p+1}}}$$

*is convergent for* $t=x$.

Lemma 1. *If the conditions of Theorem 1 are satisfied, then*

$$s_n(x) = \frac{a_0}{2} + \sum_{k=1}^{n} (a_k \cos kx + b_k \sin kx) = o(n^{\frac{1}{p+1}}).$$

1) F. T. Wang: Tohoku Math. Journ. (under the press).