PAPERS COMMUNICATED

151. *Some Remarks on a Theorem Concerning Star-shaped Representation of an Analytic Function.*

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1. The following theorem is originally due to Mr. S. Takahashi[1] and was completed by Mr. K. Noshiro.[2]

Let $$f(z)=z+a_2z^2+\cdots\cdots+a_nz^n+\cdots\cdots\ ^{3)}$$
be regular for $|z|<1$ and

$$(1)\qquad |f'(z)|<M \qquad (M>1)$$

for $|z|<1$, then the circle $|z|<\frac{1}{M}$ is represented conformally by $f(z)$ on a star-shaped domain with respect to the origin. Moreover the limiting case is attained by the function

$$f(z)=M\int_0^z\frac{1-Mz}{M-z}dz=M\left(Mz+(M^2-1)\log\left(1-\frac{z}{M}\right)\right),$$

whose derived function has a zero-point at $z=\frac{1}{M}$.

Examining the proof of Mr. Noshiro for this theorem, we see that the condition (1) can evidently be replaced by

$$(2)\qquad \left\{\frac{1}{2\pi}\int_0^{2\pi}|f'(re^{i\theta})|^2d\theta\right\}^{\frac{1}{2}}\leqq M \quad \text{for} \quad 0\leqq r<1.$$

2. Now we can naturally propose the following problems.

(I) To find the radius of the greatest circle with the center at the origin which is represented conformally by every $f(z)$ on a star-shaped domain with respect to the origin under the condition

$$(3)\qquad \left\{\frac{1}{2\pi}\int_0^{2\pi}|f(re^{i\theta})|^2d\theta\right\}^{\frac{1}{2}}<M \quad (M>1) \quad \text{for} \quad 0\leqq r<1.$$

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1) Tohoku Math. Journ. **33** (1931), p. 58.
2) Proc. **7** (1932).
3) In this note, $f(z)$ means throughout the function of this form.