139. *On the Universal Covering Group of Lie's Continuous Groups.*

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Let a system of matrices $\{U_1, U_2, \ldots\ldots, U_r\}$ form an infinitesimal group of Lie's continuous group, so that

$$[U_i, U_k] = U_i U_k - U_k U_i = \sum_{j=1}^{r} c_{ikj} U_j ,$$

and put

$$U(\omega(x)) = \sum_{k=1}^{r} \omega^k(x) U_k .$$

In the case where the constants of structure c_{ikj} are real, we consider the space $\mathfrak{F}$ of vectors $\omega(x) = (\omega^1(x), \omega^2(x), \ldots\ldots, \omega^r(x))$, whose components $\omega^k(x)$ are arbitrary analytic functions in $|x| \leqq 2$ and real functions in $-2 \leqq x \leqq 2$.

Let G be a set of the fundamental solutions[1] $Y(x)$ satisfying the differential equation of matrix

$$\text{(1)} \qquad \frac{dY(x)}{dx} = U(\omega(x))Y(x), \qquad \text{where} \qquad \omega(x) \varepsilon \mathfrak{F};$$

then we know[2] that G forms a topological group and a set of $Y(\xi)$, for a fixed point ξ, forms Lie's continuous group generated by $\{U_1, U_2, \ldots\ldots, U_r\}$.

Now, if the fundamental solutions $Z(x)$ and $Y(x)$ corresponding to $U(v(x))$ and $U(\omega(x))$ respectively, satisfy the same boundary condition $Z(1) = Y(1) = Y$, then we say that $Z(x)$ is contained in the same class $Y = \{Y(x)\}$ as $Y(x)$. Further, if the vector $v(x)$ of $Z(x)$ is deformable to the vector $\omega(x)$ of $Y(x)$ in the vector space of the above class $Y = \{Y(x)\}$, then we define that $Z(x)$ is congruent to $Y(x)$, that is, $Z(x) \equiv Y(x)$.

If $\mathfrak{G}^*$ be the group deduced by this congruence from G, we have

Theorem.[2] *$\mathfrak{G}^*$ is the universal covering group*[3] *of Lie's connected group $\mathfrak{G}$ generated by $\{U_1, U_2 \ldots\ldots, U_r\}$.*

Example. In the case where

$$U_1 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \quad \text{and} \quad U_2 = \begin{pmatrix} 0 & 0 \\ 0 & i \end{pmatrix},$$

the fundamental solution $Y(x)$ of (1) is expressed by

$$Y(x) = \begin{pmatrix} e^{\int_0^x \omega^1(t)dt} & 0 \\ 0 & e^{i\int_0^x \omega^2(t)dt} \end{pmatrix}$$

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1) If $Y(0) = E$, where E denotes the unit matrix, we call $Y(x)$ the fundamental solution of (1).