12. *On the Univalency and Multivalency of a Class of Meromorphic Functions.*

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(Comm. by S. KAKEYA, M.I.A., Feb. 12, 1936.)

1. *Theorem.*

Definition. Let z be a complex variable. We say that a domain is a *fan-shaped*, if it is given by the following expression:

$\theta_1 \leq \arg z \leq \theta_2$ (θ_1, θ_2 are two arbitrary angles such as $\theta_1 \leq \theta_2$);

$r_1 \leq |z| \leq r_2$ (r_1, r_2 are two arbitrary real numbers such as $0 \leq r_1 \leq r_2$).

We consider also as special case the figure obtained by putting $r_2 = \infty$ in the above expression.

Theorem. *Consider a function* $f(z) = \frac{a}{z} + g(z)$ *defined in a certain convex domain* A, *where* $g(z)$ *is regular in the domain* A *and* a *is an arbitrary constant. Let* p *be a positive integer. Suppose that*

(1°) $g^{(p)}(z)$ $(z \in A)$ *is contained in a convex domain* $\mathfrak{A}$,

(2°) *there exist a fan-shaped domain* B *such that the image* $\mathfrak{B}$ *of* B *transformed by the function* $w = \frac{(-1)^{p+1} p!\, a}{z^{p+1}}$ *is disjoint from* $\mathfrak{A}$: $\mathfrak{A} \cdot \mathfrak{B} = 0$. *Then* $f(z)$ *is at most* p*-valent in the common part of* A *and* B: $A \cdot B$.

Remark. Evidently, the domain $\mathfrak{B}$ is also fan-shaped and can be easily constructed from B.

Lemma. P. Montel[1] has proved the following lemma:

Be $g(z)$ *a function which is regular in a certain convex domain* A. *Let* $z_1, z_2, \dots, z_p, z_{p+1}$ *be* $p+1$ *arbitrary points of* A. *Consider the following expressions*

$$\Delta_0(z_1) = g(z_1), \qquad \Delta_1(z_2, z_1) = \frac{g(z_2) - g(z_1)}{z_2 - z_1}, \qquad \dots\dots\dots\dots,$$

$$\Delta_p(z_{p+1}, z_p, \dots, z_1) = \frac{\Delta_{p-1}(z_{p+1}, z_{p-1}, \dots, z_1) - \Delta_{p-1}(z_p, z_{p-1}, \dots, z_1)}{z_{p+1} - z_p}.$$

Then $p!\, \Delta_p(z_{p+1}, z_p, \dots, z_1) \in \mathfrak{A}$ *where* $\mathfrak{A}$ *is a convex domain which contain all the points* $g^{(p)}(z)$, $z \in A$.

Proof of the Theorem. We take $p+1$ arbirary points $z_1, z_2, \dots, z_{p+1}$ in $A \cdot B$ and we consider the following expressions $\bar{\Delta}_0, \bar{\Delta}_1, \dots, \bar{\Delta}_p$:

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1) P. Montel: Annali R. Scuola normale super. di Pisa, 2 serie, 1, 1932, p. 371–384; and Comptes Rendus, t. 201, 1935, p. 322–324.