# 56. *Certain Identities in a Generalized Space.*

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(Comm. by S. KAKEYA, M.I.A., June 12, 1936.)

1. In this paper I shall prove certain identities in a special Kawaguchi space of order $m$ and of dimensions $n$, where the curve-length

$$s=\int_{t_0}^{t} F(t, x^{(0)i}, \ldots\ldots, x^{(m)i})\,dt$$

of a curve $x^i=x^i(t)$ is invariant under transformation of $t$. I have mentioned in a previous paper[1] the intrinsic derivations along a curve for a contravariant vector $X^i$ in a Kawaguchi space:

$$\text{(A)} \qquad \overset{2m-\alpha-\rho}{D_{ij}}\,(\overset{\alpha}{E})X^j=\sum_{\beta=\rho}^{2m-\alpha}\binom{\beta}{\rho}\overset{\alpha}{E}_{i(\beta)j}X^{j(\beta-\rho)},$$

where $\overset{\alpha}{E}_i$ is a Synge vector of $\alpha$-th kind[2] and we put

$$\overset{\alpha}{E}_{i(\beta)j}=\frac{\partial \overset{\alpha}{E}_i}{\partial x^{(\beta)j}}, \qquad X^{j(\beta-\alpha)}=\frac{d^{\beta-\alpha}X^j}{dt^{\beta-\alpha}},$$

$$x^{(\beta)j}=\frac{d^\beta x^j}{dt^\beta}, \qquad x^{(0)j}=x^j.$$

Put $X^i=x^{(1)i}$ in (A), then (A) gives rise to many intrinsic vectors and these vectors may not be derived algebraically from the others and the Synge vectors in general. It should be noted, however, that these vectors are nothing but the Synge vectors except some numerical constant factors, if the curve-length $s$ is invariant under transformation of $t$.

2. Before proof of this fact, it must be remarked that there exist the Craig conditions

$$\text{(1)} \qquad \sum_{\beta=\rho}^{m}\binom{\beta}{\rho}x^{(\beta-\rho+1)i}F_{(\beta)i}=\delta^1_\rho F^{3)} \qquad (\rho=1, 2, \ldots\ldots, m),$$

if $s$ remains unaltered by transformation of $t$. As a consequence of (1) it follows

$$\text{(2)} \qquad \sum_{\beta=\rho}^{m}\binom{\beta}{\rho}x^{(\beta-\rho+1)i}F_{(\beta)i(\gamma)j}+\binom{\gamma+\rho-1}{\rho}F_{(\gamma+\rho-1)j}=\delta^1_\rho F_{(\gamma)j}.$$

Put

$$\text{(3)} \qquad \overset{\alpha}{A}_{(\mu)i}(\rho)\equiv\sum_{\lambda=\rho}^{m+\alpha}\binom{\lambda}{\rho}F_{(\mu)i}{}^{\cdot(\alpha)}_{\cdot\,(\lambda)j}x^{(\lambda-\rho+1)j} \quad \text{for} \quad \rho\geqq 1,$$

then we have on account of (2)

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1) A. Kawaguchi: Some intrinsic derivations in a generalized space, Proc. **12** (1936), 149–151.

2) See A. Kawaguchi: loc. cit.

3) H. V. Craig: On a generalized tangent vector, American Journal of Mathematics, **57** (1935), 457–462. $\delta^\alpha_\rho$ denotes the Kronecker delta, i.e. $=1$ for $\alpha=\rho$ and $=0$ for $\alpha\neq\rho$.