42. A Theorem on Operational Equation.

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1. In addition to the convections of our previous note[1] we shall here make some assumptions, and we shall prove a theorem on operational equation which corresponds to that of Schürer on the solution of linear differential equation of infinite order with constant coefficients.[2] The solutions now in our consideration correspond to those of finite grade[3] on the theory of differential equation of infinite order.

2. The assumptions which we will add to those[4] of our previous note are the following:

1°. The function-set $(C)_t$ is now defined as consisted of all elements $g(x)$ of $(B)_t$ which satisfy the boundary condition at t

$$L_t\{g(x)\}=0\,, \tag{1}$$

where L_t is a linear functional of $g(x)$. Here we assume that, for any λ of $\mathfrak{M}$, $j_\lambda(x, t_0)$ does not satisfy the boundary condition at the point t_0 that is,

$$L_{t_0}\{j_\lambda(x, t_0)\}=a_0 \neq 0\,, \tag{2}$$

where a_0 is independent of λ.

2°. In the following t_0 is fixed, and therefore we may and we shall write $j_\lambda(x)$ in stead of $j_\lambda(x, t_0)$.

3°. Let $\mathfrak{X}$ be a system of subset[5] of Y_{t_0} which constitutes a corpus.[6] For any $Y \in \mathfrak{X}$ and for any function $f(x) \in (A)_{t_0}$, we shall define a function $f_Y(x)$ which is only defined on Y and which there equals to $f(x)$. We assume that $(A)_{t_0}$ possesses the property that, for any fixed Y of $\mathfrak{X}$, the set of all $f_Y(x)$ constitutes a normalised Banach space, whose norm will be designated by $||f_Y(x)||_Y$ or simply by $||f(x)||_Y$.[7]

4°. A sequence of functions $\{f_n(x)\}$ in $(A)_{t_0}$ is said *to be a Cauchy-sequence in the generalised sense*, if, however we may choose Y from $\mathfrak{X}$,

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1) T. Kitagawa: *A* Formulation of Operational Calculus, This Proceeding, **13**. We quote this by $[F]$. See specially § 2.

2) F. Schürer: Eine gemeinsame Methode zur Behandlung gewisser Funktional gleichungsprobleme. Leipziger Berichte, vol. **70** (1918).
See specially C.L-Gleichungen hoher Ordnung p. 210.

3) See, for example, Davis: The theory of linear operators, (1936) Chapter V, Grades defined by Special Operators.

4) See $[F]$ § 2 and § 3.

5) Under a subsat of X, we understand "echte" subset.

6) Under a corpus, we understand a system of stets for which if $Y \in \mathfrak{X}$ and $Z \in \mathfrak{X}$, then $Y.\ Z$, $Y-Z$ and $Y+Z$ also belong to $\mathfrak{X}$.

7) For example, let (A) be consisted of all functions which are quarely integrable in any bounded measurable set Y of real-axis, and let

$$||f(x)||_Y \equiv ||f_Y(x)||_Y = \sqrt[2]{\int_Y |f(t)|^2 dt}$$