41. *A Formulation of Operational Calculus.*

By Tosio KITAGAWA.

Mathematical Institute, Faculty of Science, Osaka Imperial University.

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1. The object of this note is to communicate a formulation of operational calculus which is a method of solving some class of functional equations by considering allied functional operations which are permutable with certain fundamental operation as functions of the latter and thus by reducing the problems to the ordinary calculus of complex valued functions whose independent variable is real or complex number.

Our method is much suggested by a new expansion formula of Delsarte's,[1] and the results yield us a generalisation of some of our theorems on the linear translable functional equations and Cauchy's series.[2]

2. *Convections.* 1° Let us designate by small latin letters elements of an abstract space X, by large latin letters subsets[3] of X, and by large latin letters within round brackets linear set of functions defined at a subset of X.

In the following we assume that for any point t of X there correspond three sets $(A)_t$, $(B)_t$ and $(C)_t$ consisting of functions $f(x)$ defined on the set $Y_t \subset X$, and such that $(A)_t \supset (B)_t \supset (C)_t$. The class (A) is composed with all $f(x)$ which are defined over X and which belong to $(A)_t$ as functions defined on Y_t. The class (B) is composed with all $f(x)$ which belong to both (A) and the domain of the linear operation $\mathfrak{D}$.

2° The fundamental elements of $(B)_t$ are a class of functions $j_\lambda(x, t)$ which belong to $(B)_t$, and which are parametrically dependent upon λ, where λ constitutes a two dimensional simply connected domain $\mathfrak{M}$ in the complex λ-plane.

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1) J. Delsarte: [I] Sur un principle générale de developpement des fonctions d'une variable réelle en série des fonctions entières, C. R., Paris, **200** (1935).

[II] Sur l'application d'un principle général de développement des fonctions d'une variable aux séries de fonctions de Bessel, C. R., Paris, **200** (1935).

[III] Sur un procédé de développement des founctions en séries et sur quelques applications. J. Math. pures appl., IX, s. **15** 97–102 (1936).

We shall quote these papers by I, II and III respectively.

2) T. Kitagawa: On the theory of linear translatable functional equation and Cauchy's series, Japanese Journ. Math. **13** (1937). (Under press). We shall quote this paper by $[T]$.

3) Under a subset of X, we always mean an "echte" subset.

4) For example we may mention a special case defined as follows:

1° $X=(-\infty, +\infty)$

2° $Y_t=(t-a, t+a)$ $(0 < a < +\infty)$

3° $(A)_t$ is consisted of all functions defined and quarely integrable in the interval $(t-a, t+a)$.

4° $(B)_t$ is consisted of all functions defined and m-time differentiable in the interval $(t-a, t+a)$.

5° $(C)_t$ is consisted of all functions $f(x)$ which belong to $(B)_t$ and such that $f^{(m)}(t)=0$.