40. On Valiron's Theory of Linear Differential Equation of Infinite Order with Constant Coefficients.

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1. Previously[1] we have given an interpretation of the Valiron's theory of linear differential equation of infinite order with constant coefficients[2] from the standpoint of the theory of linear translatable functional equation. We wish, in the following, to complete this idea in some respect in order to bring it into a more intimate connection with the original Valiron's theory.

2. The most important theorem[3] in the Valiron's theory is the following: Let Λ be a linear differential operator of infinite order with constant coefficients whose generating function $G(\lambda)$ is an integral function of order 1 and of mean type with Valiron's condition.[4] Let $f(z)$ be a solution of the functional equation

$$\Lambda f(z)=0\,, \qquad (|z-z_0|<h) \tag{1}$$

and let it be regular in the domain $|z-z_0|<D+h$.[4']

Then a sufficient condition for that $f(x)$ is developable into its Dirichlet's series in the domain $|z-z_0|<h$ is that the series[5]

$$\sum |e^{a_n z}Q_n(z)| \tag{2}$$

should converge in the first domain $|z-z_0|<D+h$.

We will connect this theorem with our expansion-theory of Cauchy's series.[6]

3. What Ritt and Valiron called a Dirichlet series is nothing but the Cauchy's series of $f(z)$ with respect to the linear translatable operator.[7]

Its section with respect to a contour C_r is, therefore, given by

$$S_r(z, z_0; f)\equiv\frac{1}{2\pi i}\oint_{C_r}\frac{e^{\lambda z}}{G(\lambda)}\underset{0}{\Lambda_\xi}\left[e^{\lambda\xi}\int_0^\xi e^{-\lambda\eta}f(z_0+\eta)\,d\eta\right]d\lambda\,. \tag{4}$$

Now the direct computation yields us

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1) T. Kitagawa: On the theory of linear translatable functional equation and Cauchy's series, Japan. Journ. Math., **13** (1937) (Under press).

2) G. Valiron: Sur les solutions des équations différentielles linéaires d'ordre infini et a coefficients sonstants, Annales scient. l'école norm. sup., III serie, Tome **46** (1929).

3) See G. Valiron, loc. cit., Theorem XVI (p. 41).

4)-4') See G. Valiron, loc. cit., Theorem XVI (p. 41).

5) $\sum e^{a_n z}Q_n(z)$ is the Dirichlet series of $f(z)$.

6) See T. Kitagawa, loc. cit., Introduction and Chaper II, § 9.

7) See T. Kitagawa, loc. cit., Introduction, specially Definition II.

There we have defined a Cauchy's series in the real range, but it may be easily generalised to a complex domain.