38. *An Almost Periodic Function in the Mean.*

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Let x be a variable point in a measurable point set R_n of n dimensional Euclidean space. Let the function $f(t; x)$ be defined for all $x \subset R_n$ and $-\infty < t < \infty$; summable with index $p \geqq 1$ in x for all t in the sense of Lebesgue integral,[1] continuous in t; and continuous in the mean for x, that is, for any given $\varepsilon > 0$ there exists a point δ in n dimensional Euclidean space such that

$$\int_{R_n} |f(t; x+\delta) - f(t; x)|^p dx < \varepsilon^p$$

for all t.

Displacement numbers τ will be taken in the direction of the t-axis; these will be defined as follows:

We say τ is a displacement number of $f(t; x)$ belonging to ε if

$$\int_{R_n} |f(t+\tau; x) - f(t; x)|^p dx \leqq \varepsilon^p$$

uniformly for all t.

A function $f(t; x)$ is said to be almost periodic in the mean in t in any region as above if all the possible displacement numbers of $f(t; x)$ belonging to any given ε form a relatively dense set of numbers along the t-axis.

Muckenhoupt[2] and Avakian[3] have studied an almost periodic function in the mean with index 2 and applied the theory to some physical problems.

Bochner[4] has also shown that an almost periodic function in the mean with index $p \geqq 1$ has the Fourier series

$$f(t; x) \sim \sum_n A_n(x) e^{i\Lambda_n t}$$

$$\int_{R_n} |A_n(x)|^p dx < \infty$$

and proved "approximation theorem" for this class of an almost periodic function, whose enunciation is as follows:

For any almost periodic function in the mean with index $p \geqq 1$ there exists always a sequence of exponential polynomials

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1) In this paper the integral means always the Lebesgue integral.

2) Muckenhoupt, Almost Periodic Functions and Vibrating Systems. Journ. Math. Phys., Massachusetts Institute of Technology, **8** (1929), 163.

3) Avakian, Almost Periodic Functions and the Vibrating Membrane. Journ. Math. Phys., Massachusetts Institute of Technology, **14** (1935), 350.

4) Bochner, Abstrakte fastperiodische Funktionen. Acta Mathematica, **61** (1933), 149.