37. *A Note on the Singular Integral.*

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In the present paper,[1] I will give a remark about the convergence of the integral

$$(1) \qquad T_m(x;f)=(2\pi)^{-\frac{1}{2}}\int_{-\infty}^{\infty} K(x-u,\ m)f(u)du\,.$$

Mr. Northrop[2] gave the necessary and sufficient conditions in terms of Fourier transform of $K(x, m)$ for the convergence of $T_m(x;f)$ to $f(x)$ in the mean L_2 for every function $f(x)\in L_2(-\infty,\infty)$.[3] And recently he treated the same problem and has given sufficient conditions for the convergence in the mean L_q in the case where $f(x)$ is the Fourier transform in L_q of some function in L_p, and necessary conditions for the convergence in the mean L_p in the case where $K(x, m)$ is the Fourier transform in L_p of some function in L_p and $f(x)\in L_q$, where $1<p\leqq 2$ and $\frac{1}{p}+\frac{1}{q}=1$. The cases $p=1$, $q=\infty$ and $p=\infty$, $q=1$ were not treated. We here consider the case closely related to this.

H. Hahn[4] has previously given the sufficient conditions for the convergence in the mean L_1 of $\int_{-\infty}^{\infty} K(x,\ u;\ m)f(u)du$ to $f(x)\in L_1$, but not in terms of Fourier transform.

Now consider the integral

$$f(x,\ m)=\frac{1}{\pi}\int_{-\infty}^{\infty} f(u)\,\frac{\sin m(x-u)}{x-u}\,du=(2\pi)^{-\frac{1}{2}}\int_{-m}^{m} F(u)e^{ixu}du\,,$$

where $F(x)$ is the Fourier transform of $f(x)$, or $f(x)$ is the Fourier transform of $F(x)$. If $f(x)\in L_r(r)$ $(r>1)$, this converges in the mean L_r to $f(x)$. Northrop's theorem may be considered as the extension of this fact. But this fact does not hold when $f(x)\in L_1$. Therefore it will be natural to modify the mode of convergence when $f(x)\in L_1$. Concerning the above fact, I had reached the result[5] that if $f(x)\in L_1$, then

$$(2) \qquad \lim_{m\to\infty}\int_{-\infty}^{\infty}\phi\Big\{f(x,\ m)-f(x)\Big\}dx=0\,,$$

where $\phi(x)=\dfrac{|x|}{|\log|x||^{1+\varepsilon}+1}$ $(\varepsilon>0)$.

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1) My former name was Tatsuo Takahashi.

2) Northrop, Note on a singular integral, I, Bull. Amer. Math. Soc. **40** (1934); II, Duke Math. Journ., **2** (1936).

3) Hereafter we write L_p instead of $L_p(-\infty,\ \infty)$.

4) Hahn, Wiener Denkschriften, **93** (1917), 667.

5) T. Takahashi, On the conjugate function of an integrable function and Fourier series and Fourier transforms, Sci. Rep. Tôhoku Imp. Univ. Ser. I. **25** (1936).