52. *Theory of Connections in a Kawaguchi Space of Order Two.*

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In the present paper the writer will give the foundation to the geometry in a Kawaguchi space of order two and of dimension n by introducing connections under the point transformation group. An element of this space is not a point but a line element of the third order. In the space of dimension two E. Cartan has discussed already the theory of invariants under the contact transformation group.[1]

1. The metrics in the space with a point coordinate system x^i $(i=1, 2, \ldots\ldots, n)$ is given by $s=\int F(x, x', x'')dt$, putting $x'^i=\frac{dx^i}{dt}$ and $x''^i=\frac{d^2x^i}{dt^2}$. When this metrics varies by a change of parameter t the theory of connections can be easily and completely discussed. We shall assume therefore that this metrics is invariant under any change of parameter. This assumption leads to the necessary and sufficient conditions:

$$(1) \qquad F_{(2)i}x'^i=0\,, \qquad 2F_{(2)i}x''^i+F_{(1)i}x'^i=F\,.^{2)}$$

The Synge vectors $\overset{a}{E}_i\ (a=0, 1, 2)$ are not invariant under a change of parameter, but it is not difficult to find out the invariant vectors which have the forms

$$(2) \qquad \overset{0}{\mathfrak{E}}_i=F^{-1}\overset{0}{E}_i\,, \qquad \overset{1}{\mathfrak{E}}_i=\overset{1}{E}_i+F^{-1}F^{(1)}\overset{2}{E}_i\,, \qquad \overset{2}{\mathfrak{E}}_i=F\overset{2}{E}_i\,.$$

The tensor

$$(3) \qquad g_{ij}=2F^3F_{(2)i(2)j}+\overset{1}{\mathfrak{E}}_i\overset{1}{\mathfrak{E}}_j+\overset{2}{\mathfrak{E}}_i\overset{2}{\mathfrak{E}}_j$$

is also invariant and its determinant is not equal to zero, if the matrix $(2F_{(2)i(2)j}+\overset{2}{\mathfrak{E}}_i\overset{2}{\mathfrak{E}}_j)$ is of rank $n-1$, as follows from (1): $g_{ij}x'^j=-F\overset{1}{\mathfrak{E}}_i$. g_{ij} may be functions of a line element of the third order. Now we shall take this tensor as the fundamental tensor in our space and its contravariant components are defined by $g_{ij}g^{jk}=\delta_i^k$. It can be seen without difficulty that $F^3\overset{1}{\mathfrak{E}}_{i(3)j}=g_{ij}-\overset{1}{\mathfrak{E}}_i\overset{1}{\mathfrak{E}}_j$.

2. We call a quantity Q (scalar, vector, tensor, etc.) which behaves under a change of parameter $t^*=t^*(t)$ in the way:

$$Q(t^*)=a^pQ(t)\,, \qquad a=\frac{dt}{dt^*}\,,$$

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1) E. Cartan, Journal de Mathématique, (9) **15** (1936), pp. 42–69.

2) We adopt here the same notations as in the previous paper of the present author: Some intrinsic derivations in a generalized space, Proc. **12** (1936), pp. 149–151.