65. *Theory of Connections in a Kawaguchi Space of Higher Order.*

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The object of the present paper is to give the foundation to the geometry in a Kawaguchi space of order m (m: a positive integer) and of dimension n by generalization of the results in the previous paper.[1] An element of this space is a line element of not the mth order but the $(2m-1)$-th.

1. The assumption that the metrics in the space with a point coordinate system x^i $(i=1, 2, \ldots\ldots, n)$:

$$s=\int F(x, x', x'', \ldots\ldots, x^{(m)})\,dt$$

is invariant under any change of parameter t, offers the necessary and sufficient conditions:

$$\sum_{\lambda=a}^{m}\binom{\lambda}{a}F_{(\lambda)i}x^{(\lambda-a+1)i}=\delta_a^1 F, \tag{1}$$

putting $x^{(\lambda)i}=\dfrac{d^\lambda x^i}{dt^\lambda}$. Owing to (1) it can be derived from the Synge vectors $\overset{a}{E}_i (a=0, 1, \ldots\ldots, m)$ the following intrinsic vectors

$$\overset{a}{\mathfrak{E}}_i=F^{-1}\sum_{\lambda=a}^{m}\overset{\lambda}{E}_i A^a_{\lambda-a+1}, \qquad a=0, 1, \ldots\ldots, m, \tag{2}$$

where A^a_b are defined by the recurring formulae

$$A^0_1=1, \qquad A^a_b=\frac{dA^a_{b-1}}{dt}+A^{a-1}_b F,$$

$$A^1_c=F^{(c-1)}, \quad A^c_0=0, \quad A^0_d=0, \quad c=1, 2, \ldots\ldots, m;\ d=2, 3, \ldots\ldots, m.$$

We shall assume that the matrix $((mF_{(m)i(m)j}+\overset{m}{\mathfrak{E}}_i\overset{m}{\mathfrak{E}}_j))$ is of rank $n-1$, then the determinant of the intrinsic tensor

$$g_{ij}=mF^{2m-1}F_{(m)i(m)j}+\overset{m}{\mathfrak{E}}_i\overset{m}{\mathfrak{E}}_j+\overset{1}{\mathfrak{E}}_i\overset{1}{\mathfrak{E}}_j \tag{3}$$

is not identically equal to zero, for $g_{ij}x'^j=-F\overset{1}{\mathfrak{E}}_i$. g_{ij} may be functions of a line element of the $(2m-1)$-th order and this tensor can be taken as the fundamental tensor. It follows immediately

$$F^{2m-1}\overset{1}{\mathfrak{E}}_{i(2m-1)j}=g_{ij}-\overset{1}{\mathfrak{E}}_i\overset{1}{\mathfrak{E}}_j. \tag{4}$$

1) A. Kawaguchi, Theory of connections in a Kawaguchi space of order two, Proc. **13** (1937), 6. We adopt here the same notations as in this paper.