79. *On the Boundary Values of Analytic Functions.*

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I. We consider a simple closed rectifiable curve C on Gaussian plane, and we give a continuous function $f(t)$ on C. If there exists a function $F(z)$ which is analytic within C and is continuous up to the boundary C, satisfying $F(t)=f(t)$ on C, then we must have

$$\int_C f(t)\,t^m dt=0\,, \qquad m=0,1,2,\ldots\ldots \tag{1}$$

since z^m is analytic within and on C.

The equations (1) are also sufficient, under a certain condition, *for the existence of such an analytic function* $F(z)$ as above. For example, (1) is sufficient if $f(t)$ is given to be analytic along C.[1] Also it is sufficient if the curve C is analytic.[2]

The paper is devoted to prove *the sufficiency of* (*1*), *in the case where* C *has the following property* P:

To every point t on C, we can so associate a pair of opposite sectors (the sides of one sector being the elongations of the other's) of center t that 1) it varies continuously with t, 2) its radius ρ and central angle ω $(0<\omega<\pi)$ are fixed, and 3) the one sector lies within C while the other lies without C.

Any curve with *continuous tangent*, for example, evidently possesses the property P. For such curve, we can give previously any angle ω less than π, taking ρ sufficiently small, and make the sectors symmetric with respect to the tangent.

II. For proving the existence of such function $F(z)$ as above, it is sufficient to see that $F(z)$ is analytic within C and tends *uniformly* to $f(t)$ when z tends to t along the bisector of the inner sector corresponding to t. Because any point z within C which approaches to t on C should approach to t_1 on C, which is near to t and the bisector of whose corresponding sectors passes through z. So $F(z)$, approaching $f(t_1)$, will tend to $f(t)$. This is based upon the fact that the said bisector generates continuously the inner side of the curve C.

When the required function $F(z)$ should exist, it must be represented, within C, by Cauchy's integral

$$\frac{1}{2\pi i}\int_C \frac{f(t)}{t-z}\,dt\,. \tag{2}$$

So we are only to show, under the condition (1), that this integral (2) (which is evidently analytic within C) tends uniformly to $f(t_0)$ when z tends to t_0 along the said bisector at t_0.

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1) S. Kakeya, Tohoku Math. Journ., **5** (1914), p. 42.
2) J. L. Walsh, Trans. Amer. Math. Soc., **30** (1928), p. 327.