PAPERS COMMUNICATED

89. *On the Transformation Theory of Siegel's Modular Group of the n-th. Degree.*

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(Comm. by T. TAKAGI, M.I.A., Nov. 12, 1937.)

C. L. Siegel[1] has defined the modular group of the n-th. degree in the following way:

Let E and 0 be n-dimensional unit and zero matrix respectively; J the $2n$-demensional matrix defined by

$$(1) \qquad J=\begin{pmatrix} 0 & E \\ -E & 0 \end{pmatrix}.$$

Then the $2n$-demensional matrices with rational integral components, satisfying the relation $M'JM=J$ are called the (homogeneous) *modular substitutions of the n-th. degree*,[2] and the group which they form, the *modular group of the n-th. degree.*

The usual modular substitutions resp. group are the modular substitutions resp. group of the 1st. degree in this sense, and some of their classical properties are extended by Siegel to the case of the n-th. degree. I will show in the following lines, how one can found the "transformation theory" for this modular group analogously to the classical one, in defining suitably the "transformation of the degree m."

We define first the *principal congruence group mod. m* in the usual manner; namely as the group $\Gamma(m)$ formed by modular substitutions $M^{(m)}$ satisfying the congruence

$$M^{(m)}\equiv\pm\begin{pmatrix} E & 0 \\ 0 & E \end{pmatrix} \qquad \text{mod. } m\,.$$

It is obviously an invariant subgroup of the modular group $\Gamma=\Gamma(1)$. Now I give the following

Definition: The $2n$-dimensional matrices T with rational integral components satisfying the relations

$$(2) \qquad T'JT=mJ\,,$$

1) C. L. Siegel: Ueber die analytische Theorie der quadratischen Formen. I, Cf. also Krazer, Lehrbuch der Thetafunktionen.

2) Siegel calls these matrices "canonical," according to their signification in the theory of algebraic functions of the genus n. The inhomogeneous (or proper) modular substitution of the n-th. degree is the substitution bearing on the symmetrical matrices X of the dimension n: $X_1=(AX+B)\ (CX+D)^{-1}$ where A, B, C, D are n-dimensional matrices so that $M=\begin{pmatrix} A & B \\ C & D \end{pmatrix}$ is "canonical." I confine myself in the following exposition to the direct consideration of *homogeneous* substitutions, as the transit to the inhomogeneous considerations present no difficulty: one has only to take the quotient group by the invariant subgroup consisting of two elements $\pm\begin{pmatrix} E & 0 \\ 0 & E \end{pmatrix}$.