PAPERS COMMUNICATED

101. *A Theorem Concerning the Fourier Series of a Quadratically Summable Function.*

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1. Recently Mr. R. Salem[1] has proved the following theorem:

If $f(x)$ is a bounded periodic function with period 2π and its Fourier coefficients are a_n, b_n, then the following relation holds for almost all values of x,

$$\lim_{s\to 0}\left[\frac{a_0}{2}+\sum_{n=1}^{\infty}\frac{a_n\cos nx+b_n\sin nx}{1+s\sqrt{\log n}}\right]=f(x). \tag{1}$$

Actually he proved the relation (1) replacing more general sequence $\{\psi_n(s)\}$ for $\{1/(1+s\sqrt{\log n})\}$. The object of the present paper is to prove the validity of (1) under the condition that $f(x)\in L_2$, i.e. is quadratically summable. In this form the theorem says more than the well known theorem of Kolmogoroff-Seliverstoff-Plessner[2] concerning the convergence factor of the Fourier series of a quadratically summable function. But we can prove our theorem by using the theorem of Kolmogoroff-Seliverstoff-Plessner.

2. *Theorem 1. If $f(x)\in L_2$ and is periodic with period 2π and a_n, b_n are its Fourier coefficients, then the relation (1) holds for almost all values of x.*

Theorem 2. In Theorem 1, we can replace the sequence $\{1/(1+s\sqrt{\log n})\}$ by the sequence $\{\psi_n(s)\}$ which satisfies the following conditions:

1°. *$\{\psi_n(s)\}$ is the decreasing and convex sequence of positive functions, $0<s\leqq 1\,(\psi_0(s)=1)$.*

2°. $\lim_{s\to 0}\psi_n(s)=1$, (n fixed).

3°. $\lim_{n\to\infty}\psi_n(s)=0$, ($s$ fixed, >0).

4°. $\psi_n(s)=O\,(\sqrt{\log n})$, ($s$ fixed, >0).

5°. $\psi_n(s)$ has a finite number of maxima for any fixed n.

The proof of Theorem 2 is quite similar as that of Theorem 1 and so we only prove Theorem 1.

Let E_1 be the set of x such that

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1) R. Salem, Sur une méthode de sommation, valable presque partout, pour les séries de Fourier de fonction continue, Comptes Rendus, **205** (1937), pp. 14-16.

" , Sur une généralisation du procédé de sommation de Poisson, ibid., **205** (1937), pp. 311-313.

2) See, Zygmund, Trigonometrical series, Warsaw (1935), pp. 253-255.