13. *On Siegel's Modular Function of the Higher Stufe.*

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In this note we are concerned with modular functions of the degree n, of the dimension $-2r$ and of the *stufe* m, which is an extension of Eisenstein's series of the *stufe* m, due to Mr. Hecke,[1] to the case of the degree n, and deduce some of the corresponding properties.

We call Siegel's modular function of the degree n, of the dimension $-2r$, and of the *stufe* m the following function,

$$f_r(X;P_1,Q_1;m)=\sum_{\substack{P\equiv P_1\\ Q\equiv Q_1 \bmod m\\ (P,Q)_m}}\frac{1}{|PX+Q|^{2r}},^{2)}$$

where X is a symmetric matrix with a positive "imaginary part" and P_1, Q_1 form a given symmetrical pair of matrices with rational integral components and have no left common divisor, while $\sum$ sums over mod m non-associated symmetrical pair of matrices P and Q which are congruent to P_1 and Q_1 respectively and have no left common divisor.

Here we call two symmetrical pairs of matrices, P, Q and P_0, Q_0 "associated mod m" when there exists an unimodular matrix U, congruent to $\pm E$ mod m, such that the relations $P_0=UP$, $Q_0=UQ$ hold.

As in the case of Siegel's modular function of the 1st. stufe, it is absolutely and uniformly convergent when the integer $r>\frac{n(n+1)}{2}$ and represents an analytic function of X in the domain H in which X has a positive imaginary part.

The behavior under a modular substitution $M=\begin{pmatrix}A & B\\ C & D\end{pmatrix}$ is as follows. Let us complete P, Q to a modular substitution $\begin{pmatrix}P & Q\\ U & V\end{pmatrix}$, then

$$\begin{pmatrix}P & Q\\ U & V\end{pmatrix}\begin{pmatrix}A & B\\ C & D\end{pmatrix}=\begin{pmatrix}PA+QC & PB+QD\\ UA+VC & UB+VD\end{pmatrix}$$

is also a modular substitution, so that $K=PA+QC$ and $L=PB+QD$ form a symmetrical pair of matrices without a left common divisor, and

$$K\equiv K_1=P_1A+Q_1C,$$

$$L\equiv L_1=P_1B+Q_1D \qquad \bmod m\,.$$

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1) E. Hecke. Theorie der Eisensteinschen Reihen höherer Stufe and ihre Anwendung auf Funktionentheorie und Arithmetik.

2) Capital letters represent n-dimensional matrices, while small letters represent integers.