PAPERS COMMUNICATED

60. *On the Contractions of Extensors.*

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(Comm. by S. KAKEYA, M.I.A., July 12, 1938.)

It is well known that any function $\Phi(v^i, w_j)$ of the components of any two vectors, one of which v^i is contravariant and the other w_j covariant, is to be the function $\Psi(\rho)$ of only their scalar product $\rho = v^i w_i$, if it is not only a scalar but also invariant in its functional form under every regular point transformation of class ω, that is, $\Phi(v^i, w_j) = \Phi(v^r, w_s)$. This fact shows that there exists essentially one and only one scalar product of two vectors v^i, w_j.

In connection with the extended point transformation

$$x^i = x^i(x^r), \qquad x'^i = \frac{\partial x^i}{\partial x^r} x'^r, \qquad i = 1, 2, \dots, N$$

$$x''^i = \frac{\partial x^i}{\partial x^r} x''^r + \frac{\partial^2 x^i}{\partial x^r \partial x^s} x'^r x'^s, \qquad x^{(M)i} = \frac{\partial x^i}{\partial x^r} x^{(M)r} + \cdots,$$

the extensor due to H. V. Craig[1] is defined, for example, by the relationship

$$T_{s\cdot}^{\cdot ar}{}_{\beta t} = T_{j\cdot}^{\cdot \gamma i}{}_{\delta k} X^{(a)r}_{(\gamma)i} X^j_s X^{(\delta)k}_{(\beta)t} \qquad a, \beta, \gamma = 0, 1, \dots, M,$$

where we put

$$X^j_s = \frac{\partial x^j}{\partial x^s}, \qquad X^{(a)r}_{(\gamma)i} = \frac{\partial x^{(a)r}}{\partial x^{(\gamma)i}}.$$

The following relations hold for $X^{(a)r}_{(\gamma)i}$

(1) $$X^{(a)r}_{(\beta)i} X^{(\beta)i}_{(\gamma)s} = \delta^a_\gamma \delta^r_s, \qquad X^{(a)r}_{(\gamma)u} = X^{(a)r}_{(\beta)i} X^{(\beta)i}_{(\gamma)u},$$

(2) $$X^{(a)r}_{(\beta)i} = 0 \qquad \text{for} \quad a < \beta$$
$$= \binom{a}{\beta} X^{r(a-\beta)}_i \quad \text{for} \quad a \geqq \beta.^{2)}$$

Although there is only one kind of contraction $\rho = v^i w_i$ for the ordinary vectors v^i, w_j, for the extensors $V^{ai}, W_{\beta j}$ we have not one but $M+1$ kinds of contraction:

(3) $$\rho^{[a]} = \sum_{\beta=a}^{M} \binom{\beta}{a} V^{\beta-a, i} W_{\beta i}.$$

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1) H. V. Craig, On tensors relative to the extended point transformation, American Journal of Mathematics, **59** (1937), 764-774.

2) $X^{r(a-\beta)}_i$ means $\dfrac{d^{a-\beta}}{dt^{a-\beta}} X^r_i$ and we adopt the notation $F^{(a)} = \dfrac{d^a F}{dt^a}$ through this paper.