73. *Abstract Integral Equations and the Homogeneous Stochastic Process.*

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§1. *Introduction.* Let each point x of a complex Banach space $\mathfrak{B}$ represent a state (x) of a physical or a mathematical system. Consider a *temporally homogeneous stochastic process* by which the state (x) is transferred to the state (y) after the elapse of a unit time. We assume that this transition is realised by a *linear mapping* T in $\mathfrak{B}$: $y=T\cdot x$. Under their respective restrictions on T and on $\mathfrak{B}$, A. Markov, B. Hostinsky, M. Fréchet, N. Kryloff-N. Bogolioubo ff and other authors investigated the asymptotic behaviour of the n-th iterate T^n of T for large n. In the present note I intend to treat the problem by the *abstract integral equations* due to F. Riesz[1] and the theory of *resolvents* due to M. Nagumo.[2] The theorem below is a generalisation of Fréchet-Kryloff-Bogolioubo ff's theorem.[3] The lemma 1 and the lemma 3 respectively generalise the theorem of Riesz and that of Nagumo. I express my hearty thanks to S. Kakutani who kindly collaborated with me in the discussion of the present note.[4] In the next paper[5] the *mean ergodic theorem* of J. von Neumann is extended to $\mathfrak{B}$, in a way as to be applied to the problem of the homogeneous stochastic process.

§2. *The theorem.* A linear mapping T of a complex Banach space $\mathfrak{B}$ in $\mathfrak{B}$ is called a (linear) *operator* in $\mathfrak{B}$. T is called *continuous* if its *norm* (absolute value) $\|T\| = \underset{\|x\|\leq 1}{\text{l.u.b.}} \|T\cdot x\|$ is finite. A continuous operator T is called *completely continuous* if it maps the unit sphere $\|x\|\leq 1$ of $\mathfrak{B}$ on a compact point set in $\mathfrak{B}$.

Let T satisfy the following two conditions:

(1) there exists a completely continuous operator V such that $\|T-V\|<1$,

(2) there exists a constant α such that $\|T^n\|\leq\alpha$ for $n=1, 2, \ldots$.

Then we obtain the

Theorem. The proper values of T with modulus 1 are isolated proper values of finite multiplicities. Let these proper values be $\lambda_1, \lambda_2, \ldots\ldots, \lambda_k$. Then there exist completely continuous operators $T_1, T_2 \ldots\ldots, T_k$, a continuous operator S and positive constants β, ε such that

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1) Acta Math. **41** (1918), 71-98.

2) Jap. J. of Math. **13** (1936), 75-80.

3) M. Fréchet: Quart. J. of Math. **5** (1934), 106-144. N. Kryloff and N. Bogolioubo ff: C. R. Paris, **204** (1937), 1386-1388.

4) He also obtained another proof of our theorem, by virtue of the mean ergodic theorem in $\mathfrak{B}$. See the following paper of Kakutani.

5) Proc. **14** (1938), 292.