68. *Remarks on the Representation of Entire Functions of Exponential Type.*

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1. Paley and Wiener proved in their work "Fourier transforms in the complex domain, Amer. Math. Colloq. XIX" that the entire function $F(z)$ can be represented as

$$F(z)=\int_{-A}^{A} e^{izt}f(t)at \tag{1.1}$$

with $f(t)$ of L_2[1], if and only if $F(z)$ satisfies

$$F(z)=O\,(e^{A|z|}) \tag{1.2}$$

and belongs to L_2 on the real axis. Extending to L_p class Plancherel-Pólya[2] and R. P. Boas[3] have proved the following theorems:

I. *If the entire function* $F(z)$ *satisfies* (1.2) *and belongs to* L_p $(1<p\leqq 2)$ *on the real axis, then* $F(z)$ *has the representation* (1.1) *with* $f(t)$ *of* $L_{p'}(-A, A)$, p' *being conjugate to* p.

II. *If* (1.1) *holds with* $f(t)\in L_p(1<p\leqq 2)$, *then* $F(z)$ *is entire and satisfies* (1.2) *and belongs to* $L_{p'}$ *on the real axis.*

The object of the present paper is to show that we can prove the theorems in the further generalized forms by the analogous method as original Paley and Wiener's one and using some results due to Hille and Tamarkin.

2. Let $F(x)$ belong to $L_p(1\leqq p\leqq \infty)$. If there exists a function $f(x)$ of L_q $(1\leqq q<\infty)$ such that

$$\lim_{N\to\infty}\int_{-\infty}^{\infty}\left|\frac{1}{\sqrt{2\pi}}\int_{-N}^{N}F(x)e^{-iux}dx-f(u)\right|^q du=0\,, \tag{2.1}$$

then we call $f(x)$ the Fourier transform in L_q of $F(x)$. By the theory of Fourier transform, if $F(x)\in L_p(1<p\leqq 2)$, then $F(x)$ has the Fourier transform in $L_{p'}$ and $F(x)$ itself is the Fourier transform in L_p of some function in $L_{p'}$.

Thus the following theorems are generalizations of I and II.

Theorem 1. If the entire function $F(z)$ *satisfies* (1.2) *and it be-*

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1) L_p means $L_p(-\infty,\infty)$ otherwise specified.

2) Plancherel and Pólya proved actually the theorem in the case of a function of many variables. Commentarii Helv. Mat. **9** (1936–37).

3) R. P. Boas, Jr. Representations for entire functions of exponential type. Annals of Math. **39** (1938), No. 2.

Hua and Shü obtained the corresponding theorems for $f(z)$ of $H^pL^p(p>2)$ on the real axis. The class H_pL_p is the one introduced by A. C. Offord.

Cf. A. C. Offord, On Fourier transforms III. Transact. Amer. Math. Soc. Hua and Shü, On Fourier transforms in L_p in the complex domain. Journal of Math. and Phys. **15** (1936).