PAPERS COMMUNICATED

72. *Asymptotic Expansions in the Heaviside's Operational Calculus.*

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1. Let p denote d/dx and $F(z)$ be a power series $\sum a_n z^n$. Operating $F(p)p^{1/2}$ on 1 we have formally

$$\frac{1}{\sqrt{\pi x}}\left(a_0-\frac{a_1}{(2x)}+\frac{1\cdot 3\, a_2}{(2x)^2}+\cdots(-1)^n\frac{1\cdot 3\cdot 5\ldots(2n-1)a_n}{(2x)^n}+\cdots\right), \qquad (1)$$

which is in general divergent. Denoting this series by $S(x)$ and putting

$$F(p)p^{1/2}.\ 1=h(x)\,,$$

we have

$$h(x)\sim S(x)\,,$$

where $\sim$ means that $S(x)$ is an asymptotic expansion of $h(x)$. This is one of the Heaviside's rules.

To give a rigorous basis for this Heaviside's rule, Mr. Carson introduced Laplace operator and defined $h(x)$ by the relation

$$\frac{F(p)}{p^{1/2}}=\int_0^\infty e^{-pt}h(t)\,dt \qquad (2)$$

and proceeded as follows.[1)]

He defined $g(t)$ by $\quad F(p)=\int_0^\infty e^{-pt}g(t)\,dt$

and deduced $\quad a_n=\frac{(-1)^n}{n!}\int_0^\infty g(t)t^n\,dt \qquad (n=0, 1, 2, \ldots)$

by assuming the existence of $\int_0^\infty g(t)t^n\,dt$ and the termwise integrability of $\sum\frac{(-1)^n t^n}{n!}g(t)$. Then by making use of the relation

$$\frac{1}{p^{1/2}}=\int_0^\infty e^{-pt}\frac{dt}{\sqrt{\pi t}}$$

and the so-called "Faltungssatz," he deduced further

$$h(x)=\int_0^x\frac{g(t)}{\sqrt{\pi(x-t)}}\,dt$$

$$=\frac{1}{\sqrt{\pi x}}\int_0^x g(t)\left(\sum\frac{1\cdot 3\cdot 5\ldots(2n-1)}{n!\,(2x)^n}t^n\right)dt\,.$$

If the termwise integrability be assumed, it results

1) See Carson, Electric circuit theory and the operational calculus, 1926, Chap. V.