16. Boundary Values of Analytic Functions.

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I. Given a rectifiable simple Jordan curve C with length c in the Gaussian plane by the equation $t=t(s)=\xi(s)+i\eta(s)$ (ξ, η being real) where the parameter s is the arc length measured from a certain fixed point t_0 to t in the positive sense along C, so that it varies in the interval $[0, c]$, the functions $\xi(s)$ and $\eta(s)$ or $t(s)$, where we have $t(0)=t(c)$, satisfying Lipschitz condition on $[0, c]$, are absolutely continuous, and at almost every point s of that interval, have derivatives such that $\big(\xi'(s)\big)^2+\big(\eta'(s)\big)^2=|t'(s)|^2=1$.

Suppose that $f(t)=f\big(\xi(s)+i\eta(s)\big)=f\big(t(s)\big)$ is a measurable function of s defined on C. Then, by the Lebesgue integrals $\int_C f(t)dt$ and $\int_L f(t)dt=\int_{t_1}^{t_2} f(t)dt$, L being an arc $\underset{t}{E}(s_1\leqq s\leqq s_2)$ with end points $t_j=t(s_j)$ $(j=1, 2)$, we mean the Lebesgue integrals $\int_0^c f\big(t(s)\big)\, t'(s)ds$ and $\int_{s_1}^{s_2} f\big(t(s)\big)\, t'(s)ds$ respectively, where $t'(s)=\xi'(s)+i\eta'(s)$.

It may be needless to say that the integrals of this kind are the generalisation of the ordinary contour integrals of a complex variable. Let us remark here that, among others, the theorem concerning differentiation of an indefinite integral and the theorem of integration by parts remain valid also for our integrals, so that, for instance, writing $F(t)=\int_{t_0}^{t} f(t)dt$, where $t_0=t(0)$, we have, for almost all the values of s,

$$F'(t)=f(t)\,,$$

and also we have

$$\int_C \frac{f(t)dt}{t-z}=\left[\frac{F(t(s))}{t(s)-z}\right]_0^c+\int_C \frac{F(t)dt}{(t-z)^2}$$

$$=\int_C \frac{F(t)dt}{(t-z)^2}\,,$$

since $t(c)=t(0)$.

II. Let D and D' be the interior and the exterior of C respectively and we shall denote the points of D and D' by z and z' respectively. Now we shall consider the analytic function $\varphi(z)$ defined by the following integral:

$$(*) \qquad \varphi(z)=\frac{1}{2\pi i}\int_C \frac{f(t)}{t-z}\,dt\,.$$

If $f(t)$ is given continuously on C, it is evidently necessary for $\varphi(z)$ to tend to $f(t)$ as $z\rightarrow t$ for every point t of C, that we should have