# 43. On a Metric Connection Along a Curve in a Special Kawaguchi Space of Order Two.

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The foundation of the geometry in a special Kawaguchi space with the arc length $s=\int\left\{A_i(x, x')x''^i+B(x, x')\right\}^{\frac{1}{p}}dt$ has been studied by Prof. A. Kawaguchi[1) 2)].

In the present paper, it is proposed to introduce a metric connection along a curve in the same space according to Kawaguchi's theory. The same notations are adopted here as those by Prof. A. Kawaguchi.

1. In one of his previous papers Prof. A. Kawaguchi[2)] has introduced the covariant derivative by the parameter $t$, under the assumption that $p \neq 1$:

$$\delta^* v^k=\frac{dv^k}{dt}+M^k_{\cdot i}v^i+L^k_{\cdot li}x''^l v^i\,, \tag{1}$$

where $M^k_{\cdot i}$ is an object and $L^k_{\cdot li}$ is a tensor which are homogeneous of degree 1 and $-1$ with regard to the $x'^j$ respectively, and the latter is symmetric with respect to both the undersubscripts.

Let $x'^k$ be put into (1) instead of $v^k$ and the left-hand side be denoted by $\overset{\vee}{x}{}^{[2]k}$, then it is found

$$\overset{\vee}{x}{}^{[2]k}=q^k_l x''^l+M^k_{\cdot i}x'^i \tag{2}$$

putting

$$q^k_l=\delta^k_l+L^k_{\cdot li}x'^i\,,$$

which is a tensor homogeneous of degree zero with regard to the $x'^j$.

Making use of the tensor $Q^l_k$ such that $Q^l_k q^k_m=\delta^l_m$ under the assumption $p\neq\frac{5}{2}$, $|q^k_l|\neq 0$ (2) offers

$$x''^h=Q^h_k\overset{\vee}{x}{}^{[2]k}-Q^h_k M^k_{\cdot i}x'^i\,. \tag{3}$$

Putting (3) into (1), one obtains

$$\delta^* v^k=\frac{dv^k}{dt}+(M^k_{\cdot j}-L^k_{\cdot lj}Q^l_m M^m_{\cdot\cdot i}x'^i)v^j+L^k_{\cdot lj}Q^l_i\overset{\vee}{x}{}^{[2]i}v^j\,, \tag{4}$$

which defines a covariant derivative along a curve in the space.

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1) A. Kawaguchi, Geometry in an $n$-dimensional space with the arc length $s=\int\left\{A_i(x, x')x''^i+B(x, x')\right\}^{\frac{1}{p}}dt$, Transactions of the A. M. S., **44**, No. 2 (1938).

2) A. Kawaguchi, Die Geometrie des Integrals $\int(A_i x''^i+B)^{\frac{1}{p}}dt$, Proc. **12** (1936), 205–208.