42. A Symmetric Connection in an n-dimensional Kawaguchi Space.

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Introduction. Geometry in the manifold, in which the arc length s of a curve $x^i=x^i(t)$ is given by $s=\int\left\{A_i(x,x')x''^i+B(x,x')\right\}^{\frac{1}{p}}dt$, was first established by Prof. A. Kawaguchi[1]. In his work two kinds of connections C and C' are introduced. The present author proposes to introduce another connection $\mathfrak{C}$ in this manifold $K_n^{(1)}$.

§ 1 is devoted to the exposition of the various quantities in the manifold $K_n^{(1)}$ and § 2 to the establishment of the symmetric connection $\mathfrak{C}$ in our manifold. In § 3 the curvature and torsion tensors are calculated. The symbolism employed in this paper is similar to that of Prof. A. Kawaguchi[1].

1. *Exposition of the various quantities.* One starts with an n-dimensional manifold with coordinates x^i in which the arc length of a curve $x^i=x^i(t)$ is given by the integral

$$s=\int\left\{A_i(x,x')x''^i+B(x,x')\right\}^{\frac{1}{p}}dt\,. \tag{1.1}$$

It is supposed that A_i and B are analytic in a certain region of their arguments and the arc length remains unaltered by any transformation of the parameter t. The latter condition implies the following identities in x^i and x'^i:

$$\begin{cases} A_ix'^i=0\,, \\ A_{k(i)}x'^i=(p-2)A_k\,, \qquad B_{(i)}x'^i=pB\,, \end{cases} \tag{1.2}$$

where partial differentiation by x'^i and x^i is denoted with (i) and $(0)i$ respectively. Thus one concludes that the A_i is homogeneous of degree $p-2$ with regard to the x'^i and B of degree p. From the first equation (1.2) it follows, on differentiating by x'^i, that

$$A_{i(k)}x'^i=-A_k\,. \tag{1.3}$$

Now one puts

$$F=A_ix''^i+B\,, \tag{1.4}$$

and introduces the Craig vector with respect to the function F:

$$-T_i=G_{ij}x''^j+2\Gamma_i\,, \tag{1.5}$$

where

$$G_{ij}=2A_{i(j)}-A_{j(i)}\,, \qquad 2\Gamma_i=2A_{i(0)l}x'^l-B_{(i)}\,. \tag{1.6}$$

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1) A. Kawaguchi, Geometry in an n-dimensional space with the arc length $s=\int\left\{A_i(x,x')x''^i+B(x,x')\right\}^{\frac{1}{p}}dt$, Trans. Amer. Math. Soc., **44**, no. 2 (1938), 153-167.